S. M. I. T, ANKUSHPUR

ENGINEERING MECHANICS

(For 1^{ST} & 2^{ND} Semester)

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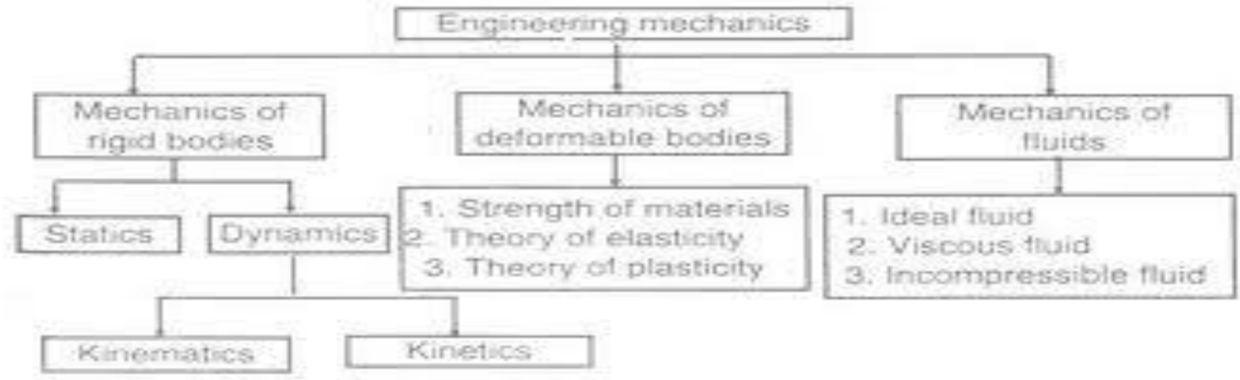
FUNDAMENTALS OF ENGINEERING MECHANICS

Engineering mechanics is that branch of applied science which deals with the laws and principles of mechanics, along with their applications to engineering problems.

Engineering Mechanics may be defined as the science, which describes & predicts the conditions of rest or motion of bodies under the action of forces.



- Classification of Mechanics – It is of 2 types –

  a. Fluid mechanics & ***b.*** Solid mechanics.

a. ***Fluid mechanics :*** It is the branch of mechanics which deals with the study of behavior of fluids (liquid or gas) either at rest or in motion.

b. ***Solid mechanics :*** It is the branch of mechanics which deals with the study of forces & their effects on rigid body.

The subject of Solid Mechanics may be sub-divided into 2 groups:-

1. Statics ***2.*** Dynamics

1. ***STATICS :*** It is the branch of engineering mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

2. ***DYNAMICS :*** It is that branch of engineering mechanics, which deals with the forces and their effects, while acting upon the bodies in motion.

Dynamics may be sub-divided into 2 branches-

a. Kinetics & ***b.*** Kinematics.

a. ***Kinetics :***

It is the branch of dynamics, which deals with the bodies in motion due to the application of forces.

b. ***Kinematics :*** It is the branch of dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

- **Fundamental Units :**

The measurement of physical quantities is one of the most important operation in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units.

All the physical quantities, met in engineering mechanics, are expressed in terms of three fundamental quantities i.e. length (L or l), mass (M or m) & time (s).

Fundamental quantities & their units :

Sl. No.	Fundamental Quantities	S.I Units
1	***Length***	***Meter (m)***
2	***Mass***	***Kilogram (kg)***
3	***Time***	***Second (s)***
4	Electric current	Ampere (A)
5	Thermodynamic Temperature	Kelvin (K)
6	Luminous Intensity	Candela (Cd)
7	Amount of Substance	Mole

- **Definitions of Fundamental S.I. Units :**
  - **a. Meter :** it may be defined as the shortest distance between two given parallel lines engraved upon the polished surface of a platinum – Iridium bar, kept at 0°C at the “International Bureau of Weights & Measures” at Serves, near Paris.

b. **Kilogram :** It may be defined as the mass of the Platinum – Iridium cylinder kept at "International Bureau of Weights & Measures" at Serves, near Paris.

c. **Second :** It is the fundamental units of time for all the four systems of units. It is $\frac{1}{24\ x\ 60x60}$ th of the mean solar day. A solar day is defined as the time interval between the instants at which the sun crosses the meridian on two consecutive days.

d. **Ampere :** It is defined as the constant current which, if maintained in two straight parallel conductors of infinite length of negligible circular cross –section & placed one meter apart in vacuum , would produce between these conductors a force of 2 x 10^{-7} Newton/meter length.

e. **Candela:** It is the luminous intensity in the perpendicular direction , of a surface of $\frac{1}{600000}$ m^2 of a black body at the temperature of freezing platinum under a pressure of 1.01325 bar.

f. **Kelvin:** It is $\frac{1}{273}$th of the absolute freezing point temperature of water .

- **Derived Units :**

Sometimes the units are also expressed in other units (which are derived from fundamental units) known as derived units, *e.g.,* the units of area, velocity, acceleration, pressure, etc.

Some Derived quantities with their units :

Derived Quantities	S.I.Units	Derived Quantities	S.I.Units
Force	N (Newton)	Velocity	m/s
Pressure	Pa (Pascal) or N/m^2(1Pa= 1 N/m^2)	Acceleration	m/s^2
Work,energy(in Joules)	1 J = 1 N-m	Angular acceleration	rad/s^2
Power (in watts)	1 W = 1 J/s	Frequency (in Hertz)	Hz
Absolute viscosity	kg/m-s	Density (mass density)	kg/m^3
Kinematic viscosity	m^2/s		

- **System of Units :**

1. **F.P.S. System :** In this system, the unit of length is foot, mass is pound & time is second. Now-a-days this system is not used.
2. **C.G.S. System :** in this system, the unit of length is centimeter (cm), mass is gram (gm) & time is second.
3. **M.K.S. System :** in this system, the unit of length is meter (m), mass is kilogram (kg) & time is second.
4. **S.I. Units :** in this system, the unit of length is meter (cm), mass is kilogram (kg) & time is second. This system of units are extensively used.

- **MASS :** It is the amount of matter contained in a given body & does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance. In S.I. units the unit of mass is kg.
- **WEIGHT :** It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with the distance of the body from the centre of the earth, therefore, the weight of the body will vary with its position on the earth's surface. It is thus obvious, that the weight is a force. In S.I. units, the unit of weight is Newton.

The relation between mass(m) & weight (W) of a body is $W = m.g$

Where, **m** in kg, **W** in Newton and **g** is the acceleration due to gravity in m/s^2

- **DIFFERENCE BETWEEN MASS & WEIGHT :**

MASS	WEIGHT
1. It is the quantity of matter contained in a body.	1. It is the force with which the body is attracted towards the centre of earth.
2. It is constant at all places.	2. It is different at different places.
3. It resists motion in the body.	3. It produces motion in the body.
4. It is a scalar quantity since it has magnitude only.	4. It is a vector quantity since it has magnitude as well as direction.
5. It can be measured by an ordinary balance.	5. It is measured by a spring balance.
6. It is never zero.	6. It is zero at the centre of earth.
7. It is measured in kilogram (kg) in M.K.S system of units as well as in S.I. units.	7. It is measured in kilogram weight (kg.wt or kgf) in M.K.S system of units & in Newton (N) in S.I. units.

F O R C E

***FORCE** : Force is an external agent by application of which, changes or tends to change the state of rest or of uniform motion of a body along a straight line.*

It is a vector quantity. Force is measured by the product of mass & acceleration. F = m X a

Where, m = mass of the body & a = acceleration of the body.

It's units are -

In C.G.S. system – gm.cm/sec^2 or gram-force (gmf) or dyne

In M.K.S system—kg.m/sec^2 or kilogram-force (kgf)

In S.I. system –Newton (N) or kilo-Newton(KN)

- **Gramforce (gmf) :**

One gram-force is that force which, when acting on a mass of one gram, produces in it an acceleration of one cm/sec^2. i.e. 1 gmf = 1 gm x 1 cm/s^2

- ***Newton :***

One newton is that force which, when acting on a mass of one kilogram, produces in it an acceleration of one meter/second^2. i.e. 1 Newton = 1 kg x 1 m/s^2

- ***Kilogram-force (kgf) :***

One kgf is that force which, when acting on a mass of one kilogram, produces in it an acceleration of 9.81m/sec^2. i.e. 1 kgf = 1 kg x 9.81 m/s^2

- ***Relation between kgf and Newton ?***

1 kgf = 1kg X 9.81m/sec^2 = (1kg X 1m/sec^2) X 9.81

= 1 Newton X 9.81 = 9.81 Newton

i.e **1 kgf = 9.81 Newton**

- **Relation between Newton & Dyne :**

1 Newton = 1 kg x 1 m/s^2 = 1000 gm x 100 cm/s^2

= 10^5 gm x m/s^2

1 Newton = 10^5 dyne.

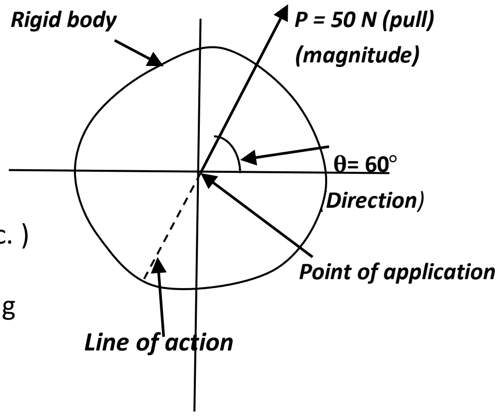
- ***Effects of forces :***

a. It may change the state of rest of a body i.e if a body is at rest, then force will set the body in motion.

b. Force may accelerate a moving body.

c. Force may retard the motion of the body.

d. It may give rise to internal stresses inside the body.



- ***Characteristics of force :***

a. Magnitude of force (i.e 10N,20kgf etc.)

b. The direction of line, along which the force acts (along OX axis, towards north, towards south-west etc.) It is also known as line of action of forces.

c. Nature of forces (push or pull).This is denoted by placing an arrow head on the line of action of force.

d. The point at which, the force acts on the body.

- ***Types of force :***

There are many kinds of force, such as gravity force, push or pull, gravitational attraction between sun & planet, tractive force of a locomotive, force of magnetic attraction, steam or gas pressure in a cylinder, wind pressure, atmospheric pressure, frictional resistance, weight, hydro-static pressure & earth pressure etc.

- ***Particle –***

A body whose dimensions are negligible when compared with the distances or the length involved in the discussion of its motion is called a particle. (A particle is a point mass or a material point)

- ***Rigid Body –***

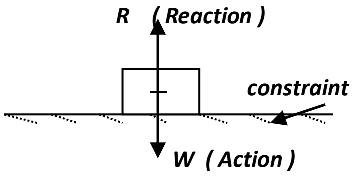
A body which doesn't deforms (i.e. retains iys shape & size) under the action of external forces is known a rigid body.

It represents the definite amount of matter, the parts of which are fixed in position relative to one another (the distance between any two points remains the same for all the times).

Actually, the bodies are not rigid, they deform under force. If the deformation is negligible when compared with the size of the body, it is assumed to be rigid.

- ***Representation of force :***
 Graphically a force may be represented by the segment of a straight line. The straight line of the force & its length represents its magnitude.
The direction of the force is indicated by placing an arrow head on this straight line. The direction of force OA is from O (i.e. starting point) to A (i.e ending point).
Either the head or tail may be used to indicate the point of application of a force.
- ***Principle of Transmissibility –***
 It states that*, if a force acting at a given point of the rigid body, is replaced by a force of same magnitude & direction on any point which is on the line of action of the force, the external effect of the force on body remains unchanged*.
- ***Principle of Superposition*** –
 When two forces are in equilibrium (equal, opposite & collinear), their resultant is zero (0) & their combined action in a rigid body is equivalent to that of no force at all. This gives principle of statics which is called as *law of superposition*.
- ***Law of superposition*** - *The action of a given system of forces on a rigid body will in no way be changed if we add or subtract from them another system of forces in equilibrium*.

Let a rigid body is acted upon by a concurrent coplanar system of forces P & Q acting at 'O' at an angle 'α'.
 Thus R is the resultant of P & Q, whose magnitude is R $= \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$. Another two sets of equilibrium forces S-S & T-T are added & subtracted to that system, which never hampers the state of the body. That means the resultant of P & Q remains altered.
- ***Constraint, Action & Reaction –***
 A body is not always free to move or rotate in all directions. The restriction to the free motion of a body in any direction is called a constraint (or support).



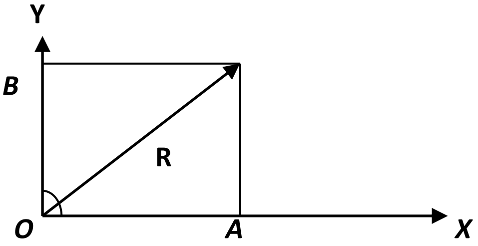
 For example – A body is resting on a horizontal plane is free to move in horizontal & vertically upward direction, but it can't move vertically downward. Hence the horizontal plane is a support which restricts or constraints the motion of the body vertically downward.
 The weight of the body on the support is action. The action of a constrained body on any support induces an equal & opposite reaction from the support. This reaction will be induced in a direction in which the support restricts the motion of the body.
 Action is a force which acts at the support by a body. As a result, the support will exert an equal force in opposite direction on the body at the point of contact is known as *reaction.*
Law of action & reaction – *Any force on a support causes an equal & opposite force from the support, so that action & reactions are two equal & opposite forces.*
- ***Tension & Compression –***
 It is two equal & opposite forces act on the axis of the rigid body when its weight is neglected, that is why it is called as *axial force*. Generally tensions are shown by arrow heads away from the joint (ends), while compression are shown towards the joint.
 Tension is a pull to which a rope or wire or rod is subjected. When a hinged rope is pulled by a force 'W', at that time the hinged point exert an equal amount of force 'W' to the body, in upward direction. This upward force on the rope is the *tension* of the rope.
 Compression is a push to which a spring, struts or rod (compressive members) is subjected.
- ***Pull & Push–*** *Pull* is a force applied to a body at its front end to move the body in the direction of the force applied. *Push* is a force applied to a body at its back end in order to move the body in the direction of the force applied.
- ***External force & Internal force –*** When a force is applied externally to a body, then that force is called *external force*. *Internal force* is that force which is set up in a body caused by the external force.
- ***Free Body –*** It is the body isolated from all other members which are connected to this body. Thus in a structure, we may consider a no. of free bodies.
- ***Free body diagram –***The drawing of an isolated body (i.e. free body) which shows the external forces in the body & the reactions exerted on it by the removed elements.
 In other words, the diagram of the isolated element or a portion of the body along with the net effects of the system on it is called 'Free Body Diagram'.

RESOLUTION OF FORCE

- ***Resolution of force :-***

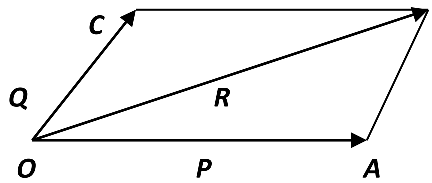
The replacement of a given force by several components which will be equivalent in action to the given force is known as *resolution of force*.

A force is generally resolved mutually in perpendicular direction.
Consider a force 'R' acting at any point
The force 'R' can be resolved in X & Y two mutually perpendicular direction.



'R' has 2 components – OA in X direction
OB in Y direction
OA & AB are called components of force 'R'
OA = horizontal component of force 'R'
OB = vertical component of force 'R'

- ***Components of a force –***



If 'R' is the resultant of two forces 'P' & 'Q', it means forces 'P' & 'Q' can be replaced by 'R'. Similarly, 'R' can be replaced by two forces 'P' & 'Q' whose joint effect on a body will be the same as 'R' on the body. Then these two forces 'P' & 'Q' are called components of 'R'. All resolved parts are components of force, but components of a force are not resolved parts of a force.

- ***Resolved parts of a force –***

Resolved parts are components of a force in two mutually perpendicular directions. Forces 'P' & 'Q' are the resolved parts of force 'R' along OA & OC respectively, where $\angle$ AOC = 90°.

- ***Resolution of a given force into two components in two assigned directions :***

Let P be the given force represented in magnitude & direction by OB as shown in fig. Also let OX & OY be two given directions along which the components of P are to be found out.

Let $\angle BOX = \alpha$ & $\angle BOY = \beta$

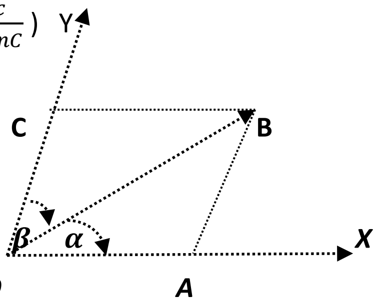
From B, lines BA & CB are drawn parallel to OY & OX respectively. Then the required components of the given force P along OX & OY are represented in magnitude & direction by OA & OC respectively.

Since AB is parallel to OC, $\angle BAX = \angle AOC = \alpha + \beta$

$\therefore \angle OAB = 180° - (\alpha + \beta)$

Now in ΔOAB, by applying sine rule, we get –

$$\frac{OA}{sin\angle OBA} = \frac{AB}{sin\angle AOB} = \frac{OB}{sin\angle OAB} \qquad \left(\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}\right)$$



or $$\frac{OA}{sin\beta} = \frac{AB}{sin\alpha} = \frac{OB}{\sin[180° - (\alpha + \beta)]}$$

or $$\frac{OA}{sin\beta} = \frac{AB}{sin\alpha} = \frac{OB}{\sin(\alpha + \beta)} \qquad (\text{ as } \sin(180° - \theta) = \sin\theta)$$

or $$\boldsymbol{OA = \frac{Psin\beta}{sin(\alpha + \beta)}} \qquad \& \qquad AB = \frac{Psin\alpha}{\sin(\alpha + \beta)}$$

But AB = OC $\qquad \therefore$ $$\boldsymbol{OC = \frac{Psin\alpha}{sin(\alpha + \beta)}}$$

- ***Difference between components & resolved parts :***

1. When a force is resolved into two parts along two mutually perpendicular directions, the parts along these directions are called resolved parts.
   But when a force is split into two parts along two assigned direction, these parts are called components of the force.
2. All resolved parts are components, but all components are not resolved parts.
3. The resolved part of a force in a given direction represents the whole effect of the force in that direction.
   But the component of a force in a given direction does not represent the whole effect of the force in that direction.

- ***Principles of resolution :-***

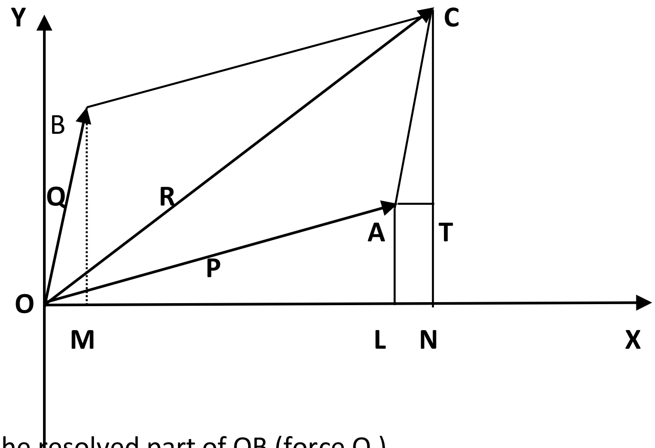
It states that, *"The algebraic sum of the resolved parts of a number of forces in a given direction is equal to the resolved part of their resultant in same direction."*

PROOF:-

Lets consider 2 forces P & Q Represented in magnitude & direction by 2 adjacent sides OA & OB of a Parallelogram.

The resultant 'R' of P & Q will be the diagonal OC of the parallelogram.

Let OX be the given direction, in which forces are to be resolved.



Now draw AL, BM & CN perpendiculars from the point A, B & C on

Also draw perpendicular AT from point A on CN.

In the triangle OBM & ACT,

OB & AC are parallel & equal in magnitude.

OM & AT are also parallel, BM & CT are parallel and BM = CT, Therefore, OM =AT = L

From the geometry of figure,

ON =OL + LN = OL + OM (as OM = LN)

But ON is the resolved part of OC (resultant R)

OL is the resolved part of OA (force P) & OM is the resolved part of OB (force Q)

Hence, resolved part of 'R' along OX = resolved part of 'P' along OX + resolved part of 'Q' along OX

(*This principle is extended for any no. of forces*.)

- ***Finding resultant of forces by 'Method of resolution'.***

The resultant of force of a given system of force, may be found out by the method of resolution as follows-

a. Resolve all the forces vertically (along OY) & find the algebraic sum of all vertical components (ΣV).
i.e. $\Sigma V = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3$

b. Resolve all the forces horizontally (along OX) & find the algebraic sum of all horizontal components (ΣH)
i.e. $\Sigma H = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3$

c. The resultant 'R' of the given forces will be given by $\mathbf{R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}}$

d. The resultant force will be inclined at an angle α with the horizontal, such that $\mathbf{Tan\ \alpha = \frac{\Sigma V}{\Sigma H}}$

➢ For equilibrium, The resultant force, R = 0. As $\mathbf{R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}}$, $\mathbf{\Sigma H = 0}$ **&** $\mathbf{\Sigma V = 0.}$
i.e. *sum of resolved part of all forces along horizontal (in x-axis) & along vertically (in y-axis) is zero*.

➢ The value of 'α' will varies as follows-

I. When ΣV = +ve, $\alpha = 0^o - 180^o$
ΣV = - ve, $\alpha = 180^o - 360^o$

II. When ΣH = +ve, $\alpha = 0^o - 90^o$ & $270^o - 360^o$
ΣH = -ve, $\alpha = 90^o - 270^o$

MOMENT OF FORCE

➢ *MOMENTS*

In case of forces, the effects of forces, acting on a body through their lines of action or at the point or at the point of their intersection. But in case of moments, the effects of forces, at some other point, away from the point of intersection or their lines of action.

- ***MOMENT OF FORCE :-***

It is the turning effect produced by a force, on the body about any point other than a point on line of action of the force.

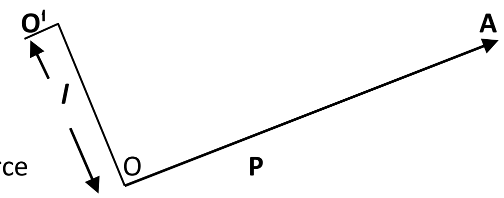
The moment of force may be defined as the product of the force & the perpendicular distance of the point through which moment is found and the line of action of force.

Mathematically,

Moment, M = P . l

Where, P = Force acting on body

l = Perpendicular distance between the point, about which the moment is reqd. and the line of action of force



Moment of force 'P' about O' = P . l = P . OO'

Where, OO' = Perpendicular distance between O' and line of action of force P = l

- ***UNITS OF MOMENT :-***

The moment of a force is the product of force and the distance, therefore the units of the moment depends upon unit of force & unit of distance.

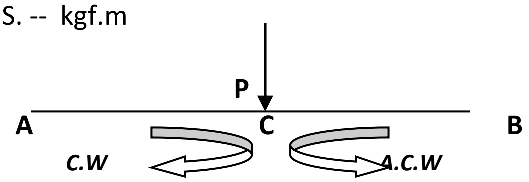
Units of moment – In C.G.S. -- gmf.cm , In M.K.S. -- kgf.m

In S.I. -- N.mm or KN.m etc.

- ***TYPES OF MOMENT :-***

I. <u>CLOCKWISE MOMENT</u> –

It is the moment of force, whose effect is to turn or rotate, the body in the same directions of hands of a clock.



II. <u>ANTICLOCKWISE MOMENT</u> –

It is the moment of force, whose effect is to turn or rotate the body in the opposite directions of hands of a clock.

Moment of force 'P' about A is 'clockwise moment'.

Moment of force 'P' about B is anticlockwise moment.

In order to distinguish turning tendency in the clockwise direction from that in the anti-clockwise direction, it has become necessary to treat moment in one direction as positive & moment in the reverse direction as negative. ***There is no hard & fast rule regarding sign convention of moments***.

- ***GRAPHICAL REPRESENTATION OF MOMENT :-***

Consider a force 'P' represented in magnitude & direction by the line AB.

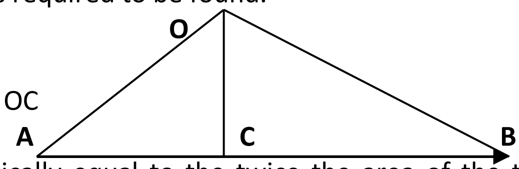
Let 'O' be a point, about which the moment of the force is required to be found.

Draw a perpendicular OC to AB . Join OA & OB.

Moment of force 'P' about O = Force P X OC

= AB X OC = 2 X ½ X AB X OC

= 2 X (Area of Δ OAB)



Thus moment of a force, at any point is geometrically equal to the twice the area of the triangle, whose base is the line representing the force & whose vertex is the point, about which the moment is taken.

- ***PRINCIPLE OF MOMENTS :***

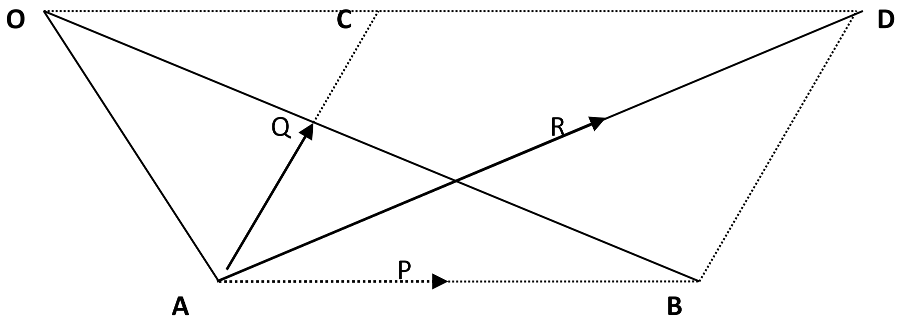
The algebraic sum of moments of all coplanar forces acting on a rigid body in equilibrium about any moment center is zero.

Mathematically, **Σ M = 0.** i.e. clockwise moments = anti-clockwise moments.

- **VARIGNON'S THEOREM :-**

It states that, ***"If a no. of coplanar forces are acting simultaneously on a particle, the algebraic sum of all the forces about any point is equal to the moment of their resultant force about the same point."***

PROOF :- Two concurrent forces P & Q represented in magnitude & direction by AB & AC respectively.



Complete the parallelogram ABDC. Join AD, which will be the resultant of P & Q i.e R.

Let 'O' be any point lying in the plane of forces P & Q about which moment is to be found.

Through point O lets draw line OD passing C (i.e O-C-D a line)

Moment of force 'P' about 'O' = 2 X Area of Δ AOB

Moment of force 'Q' about 'O' = 2 X Area of Δ AOC

Moment of force 'R' about 'O' = 2 X Area of Δ AOD

From the geometry of figure, Area of Δ AOD = Area of Δ AOC + Area of Δ ACD

But Area of Δ ACD = Area of Δ ABD = Area of Δ AOB

Therefore, Area of Δ AOD = Area of Δ AOC + Area of Δ AOB

2 X Area of Δ AOD = 2X Area of Δ AOC + 2 X Area of Δ AOB

Hence***, Moment of 'R' about 'O' = Moment of 'P' about 'O' + Moment of 'Q' about 'O'***

(This principle can be applicable for any no. of forces .)

- **ENGINEERING APPLICATIONS OF MOMENT :**
  Some of the important engineering applications of moment are given below –
  1. The levers,
  2. The balance,
  3. The common steel yard,
  4. Lever safety valve,
  5. Bell crank lever,
  6. Cranked lever etc.
- ***CONDITION FOR EQUILIBRIUM OF COPLANAR NON-CONCURRENT FORCES :-***

➢ The algebraic sum of horizontal components of all the forces & vertical components of all the forces must be zero (0)

As $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 0$, Which implies $\Sigma H = 0$ & $\Sigma V = 0$.

➢ For rotational equilibrium the resultant moment , component of all forces must be zero s i.e $\Sigma M = 0$

➢ COUPLE :

If two equal & opposite parallel forces act on a body, they don't have any resultant forces parallel to given forces. No single force can replace two equal & opposite forces, whose lines of action are different. Such a set of two equal & opposite forces, whose lines of action are different, form a couple.

A couple is thus, unable to produce any translator motion (i.e. motion in a straight line), but a couple produces rotation in the body on which it acts.

Example : forces applied to the key of clock, in winding it up & open or close the cap of a bottle etc.

➢ **MOMENT OF COUPLE :**

Two unlike parallel forces when set upon a body forms couple. The moment of the couple is the product of the force & the arm of the couple.

Mathematically, Moment of couple = P.a

Where, P = force & a = Perpendicular distance between 2 unlike parallel forces = arm of couple.

➢ **CLASSIFICATION OF COUPLES :**

Depending upon their directions, in which couple tends to rotate the body, the couples may be classified into 2 types –

a. **Clockwise couple :** A couple whose tendency is to rotate the body in a clockwise direction is called clockwise couple. It is also called ' ***positive couple*** '.

b. **Anti-clockwise couple :** A couple whose tendency is to rotate the body in an anti-clockwise direction is called anti-clockwise couple. It is also called ' ***negative couple*** '.

There is no hard & fast rule regarding sign convention of couples.

➢ **CHARACTERISTICS OF COUPLE :**

A couple (whether clockwise or anti-clockwise) has the following characteristics –

I. The algebraic sum of the forces, constituting the couple is zero (0)

II. The algebraic sum of the moments of the forces constituting the couple about any point is the same, & equal to the moment of couple itself.

III. A couple cannot be balanced by a single force, but can be balanced by a couple, but of opposite sense (i.e. equal magnitude, opposite in sign & coplanar in action with given couple).

IV. Any no. of coplanar couples can be reduced to a single couple (Resultant couple), whose magnitude will be equal to the algebraic sum of the moment of all couples.

V. The action of a couple in a body doesn't change if we change both the magnitudes of the forces & the arm of the couple in such a way that the moment of couple remains unchanged.

VI. A system of couple acting in one plane is in equilibrium if the algebraic sum of their moments is equal to zero.

FORCE SYSTEM

- ***System of forces :***
  When 2 or more forces act on a body, they are called ' *system of forces*'.
- ***Classification of forces :***
  The classification of forces are based on the point, lines of action and plane are as follows –

a. **CO-PLANAR FORCES :-**
The forces whose lines of action lie on the same plane, are known as '*coplanar forces*'.

b. **COLLINEAR FORCES :-**
The forces whose lines of action lie on the same line, are known as ' *collinear forces* '.

c. **CONCURRENT FORCES :-** The forces which meet at one point.

d. **COPLANAR CONCURRENT FORCES :-**The forces which meet at one point and the lines of action also lie on the same plane, are known as '*coplanar concurrent forces*'.

e. **COPLANAR NON-CONCURRENT FORCES :-** The forces which don't meet at a point, but their lines of action lie on the same plane, are known as '*coplanar non-concurrent forces'.*

f. **NON-COPLANAR CONCURRENT FORCES:-**
The forces which meet at one point but their lines of action don't lies on the same plane are known as '*non-coplanar concurrent forces*'.

g. **NON-COPLANAR NON-CONCURRENT FORCES :-**
The forces which don't meet at a point and their lines of action don't lies in the same plane, are known as '*non-coplanar non-concurrent forces*'.

COMPOSITION OF FORCES

- ***Resultant Force :*** If a no. of forces are acting simultaneously on a particle, it is possible to find out a single force which can replace them and it will produce the same effect as produced by all of the forces. This single force is known as *resultant of force*.

Consider a body in which 3 forces P,Q and S are acting.

If we shall replace all the forces (system of force) by a single force 'R', whose effect is same as P,Q & S, then, R= resultant of forces P,Q & S.

- ***What are the methods for finding 'resultant of forces' ?***

Generally 2 methods are employed for find out resultant of forces---

 a. Analytical method &
 b. Graphical method.

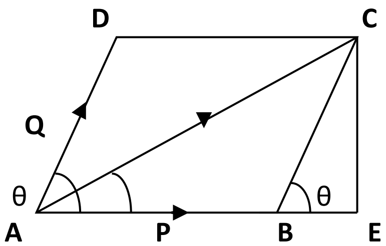
a. Analytical method is of 2 types--
 I. Parallelogram law of forces &
 II. Method of resolution.

b. Graphical method is of 2 types –
 I. Triangle law of forces &
 II. Polygon law of forces.

➢ **Analytical Method :**

- ***Parallelogram law of forces :***

It states that, *"if 2 forces acting simultaneously on a particle be represented in magnitude and direction by 2 adjacent sides of a parallelogram, their resultant may be represented in magnitude and direction by the diagonal of the parallelogram passing through the point of intersection."*



- ***Determination of Resultant of 2 forces*** :-

P & Q are two forces whose resultant is required to be found

In the parallelogram ABCD ,

Force P & Q are represented as two adjacent sides of ABCD as AB & AD respectively.

Force R = resultant force of P & Q

Draw a perpendicular CE to the extension of AB, which meets at E.

θ = angle between the forces P & Q

From the triangle AEC ,

$AC^2 = AE^2 + CE^2$

$AC^2 = (AB + BE)^2 + CE^2$ …………………(1)

From the triangle BEC, $\angle CBE = \angle DAB = \theta$

BC = force Q & $\angle BEC = 90^o$

Therefore, $BE = Q\cos\theta$ & $CE = Q\sin\theta$

By putting the values of BE & CE in equation (1), we get

$R^2 = (P + Q\cos\theta)^2 + Q^2\sin^2\theta$

$= (P^2 + Q^2\cos^2\theta + 2PQ\cos\theta) + Q^2\sin^2\theta$

$= P^2 + Q^2\cos^2\theta + Q^2\sin^2\theta + 2PQ\cos\theta$

$= P^2 + Q^2 + 2PQ\cos\theta$ $\quad\because (\sin^2\theta + \cos^2\theta = 1)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Let α = angle between resultant force R & horizontal force (i.e. P)

From the triangle ACE,

$\tan\alpha = \frac{CE}{AE} = \frac{CE}{(AB + BE)} = \frac{Q\sin\theta}{P + Q\cos\theta}$ i.e. $\alpha = \tan^{-1}\frac{Q\sin\theta}{P + Q\cos\theta}$

When $\theta = 0^o, R = P + Q$ & $\theta = 180^o, R = P \sim Q$ & $\theta = 90^o, R = \sqrt{P^2 + Q^2}$

- **When forces are equal i.e. P = Q,** $R = 2P\cos\left(\frac{\theta}{2}\right)$

- **Graphical Method** :

- **GENERAL LAWS FOR RESULTANT FORCE :-**

  The resultant force, of a given system of forces, may also be found out by following laws –

  a. Triangle law of forces, &

  b. Polygon law of forces.

a. **TRIANGLE LAW OF FORCES:-**

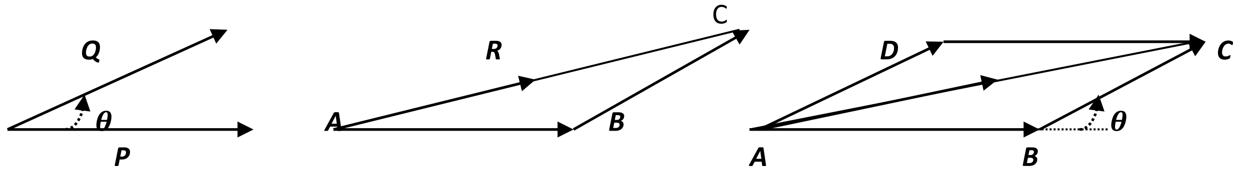
It states that,

"If two forces acting simultaneously on a particle, be represented in magnitude and direction by the 2 sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the 3rd side of triangle, taken in opposite order."

Explanation :

Let two forces P & Q acting at 'O' be such that they can be represented in magnitude & direction by the sides AB & BC of the triangle ABC.

Then according to theorem of triangle of forces, their resultant 'R' will be represented in magnitude & direction by 'AC' which is the third side of the triangle ABC taken in reverse order.



PROOF :

The parallelogram ABCD is completed with sides AB & BC of the triangle ABC. Side AD is equal & parallel to BC.

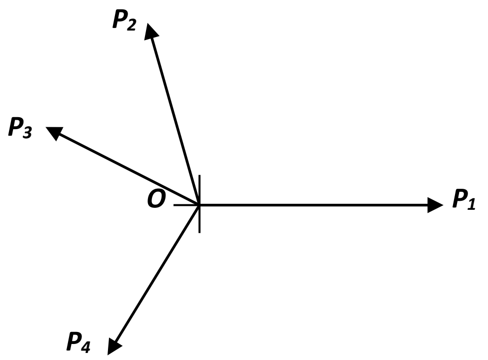
So, force 'Q' is also represented in magnitude & direction by AD. Now, the resultant of P & Q is represented in magnitude & direction by the diagonal 'AC' of parallelogram ABCD. Thus, the resultant of P & Q is represented in magnitude & direction by the third side AC of triangle ABC, taken in reverse order. Hence, triangle law of forces proved.

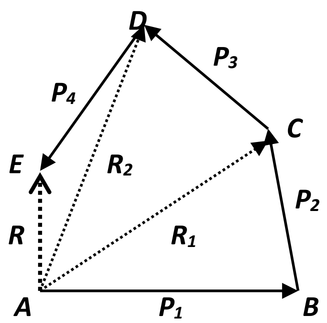
b. **POLYGON LAW OF FORCES :-**

It is an extension of Triangle law of forces for more than 2 forces. It states that,

"If a no. of forces acting simultaneously on a particle, be represented in magnitude & direction, by the sides of a polygon taken in order, then the resultant of all the forces may be represented in magnitude & direction, by the closing side of the polygon, taken in opposite order."

PROOF :





Let forces P_1, P_2, P_3 & P_4 acting at point 'O', be such that they can be represented in magnitude & direction by the sides AB, BC, CD & DE of a polygon ABCDE.

We have to prove, the resultant of these force is represented in magnitude & direction by side 'AE'.

Join 'AC' & 'AD'. According to triangle law of forces,

In Δ ABC, 'AC' represents the resultant of P_1 & P_2 i.e. R_1.

In Δ ACD, 'AD' represents the resultant of 'AC' & P_3 i.e. resultant of P_1, P_2 & P_3 i.e. R_2.

In Δ ADE, 'AE' represents the resultant of 'AD' & P_4 i.e. resultant of P_1, P_2, P_3 & P_4 i.e. R.

Hence, polygon law of forces proved.

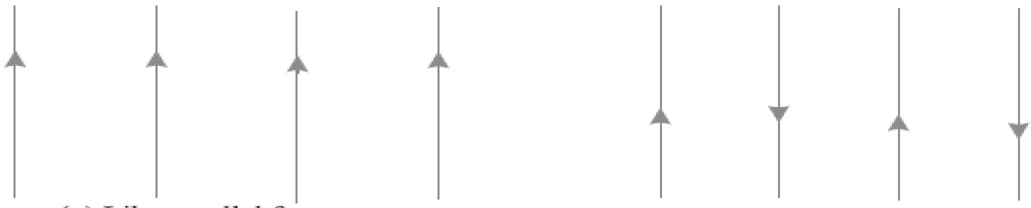
- **PARALLEL FORCES** :

  The forces, whose lines of action are parallel to each other, are known as **'parallel forces'**.

Depending upon their directions, the parallel forces may be classified into 2 categories –

a. Like parallel forces & ***b.*** Unlike parallel forces

a. ***LIKE PARALLEL FORCES –*** The forces, whose lines of action are parallel to each other, and all of them act in the same direction.



(a) Like parallel forces (b) Unlike parallel forces.

b. ***UNLIKE PARALLEL FORCES –***The forces, whose lines of action are parallel to each other and all of them don't act in the same directions, are known as unlike parallel forces.

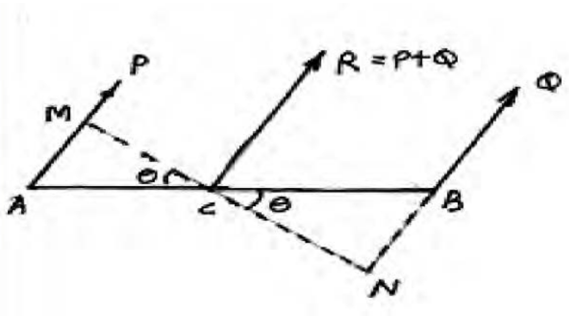
- ***Methods for finding magnitude & position of resultant force –***

The magnitude & direction of the resultant force, of a given system of parallel forces may be found out by – ***a.*** Analytical method & ***b.*** Graphical method.

<u>ANALYTICAL METHOD</u> –

- ***Resultant of 2 like parallel forces –***

Consider 2 like parallel forces 'P' & 'Q' whose resultant is to be found out acting at 'A' & 'B' and their resultant 'R' cut AB at C. By resolving parallel to P or Q, we find that R = P + Q and that R is parallel to P & Q. The moment of R about C is zero, so that the algebraic sum of moments of P & Q about C must also be zero.



Through C draw MN perpendicular to P & Q,

Then CM = AC cosθ

CN = BC cosθ

Taking moments about C, P X CM = Q X CN

P X AC cosθ = Q X BC cosθ

P X AC = Q X BC

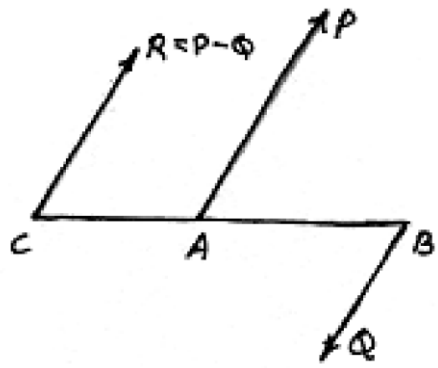
Or $\frac{P}{BC} = \frac{Q}{AC}$

∴ C divides AB internally in the inverse ratio of the forces.

- *This principle may be extended for any no. of forces.*

- ***Resultant of two unlike parallel forces -***

Let us consider two unlike parallel forces P & Q whose resultant is to be found out, acting at A & B, and their resultant R meet AB at C.



Let P be greater than Q. By resolving parallel to P or Q ,
we get R = P – Q. acting in the same sense as P.
The algebraic sum of the moments of P & Q about C
must be zero so that these moments must be equal
and opposite. Hence C must lie outside Ab,
and it must be nearer to A than to B.
Taking moments about c, we get

P X AC = Q X BC or $\frac{P}{BC} = \frac{Q}{AC}$

∴ C divides AB externally in the inverse ratio of the forces.

The point C is called the center of parallel forces. It is clear that the position of C is independent of the inclination of the forces to AB.

- *This principle may be extended for any no. of forces.*

- ***POSSIBLE CASES FOR TWO PARALLEL FORCES :***

There are 3 possible cases for 2 parallel forces –

- ***a.*** Two given parallel forces act in the same direction.
- ***b.*** They act in opposite direction & are unequal magnitude.
- ***c.*** They act in opposite direction & are equal magnitude.

a. Resultant of the two given parallel forces acting in the same direction is equal to their sum (i.e. R = P + Q). It acts along the line parallel to the lines of action of the given forces & dividing the distance between their point of application in the ratio inversely proportional to their magnitude.

b. The resultant of two unequal parallel forces acting in opposite direction is equal to the subtraction of smaller force from larger force (i.e. R = P-Q). The direction of the resultant force is acting in the direction of larger force. The distances of the components from the line of action of the resultant force are inversely proportional to their magnitudes. The line of action of the resultant force, lies outside the space between the components on the side of the larger force.

c. A system of two equal parallel forces acting in opposite directions can't be reduced to one resultant force (resultant of such force is zero). Two such forces are called a *couple*, the plane in which they act is called *the plane of couple*, & the distance between their lines of action is called *arm of the couple*.

EQUILLIBRIUM OF FORCES

- ***Equilibrium of forces :***

Two concurrent forces can be equilibrium only if their resultant is zero(0).

Two concurrent forces can be in equilibrium only if they are equal in magnitude and opposite in direction and collinear in action. In case system of forces, we have, ΣF = 0 ………for equilibrium.

- ***Equilibrant :***

When a body is subjected to a no. of concurrent forces, it moves in the direction of resultant force with uniform acceleration. However, if another force which is equal in magnitude of the resultant but opposite in direction is applied to the body, the body comes to rest.

Hence, *equilibrant of a system of force is a single force, which is equal, opposite & collinear to the resultant of the given system of forces.*

- ***METHOD OF EQUILIBRIUM OF FORCES :***

Method of equilibrium of forces are of two types –

a. Analytical method & b. Graphical method.

a. Analytical Method –

There are many analytical methods out of which few methods are as follows –

- ***I.*** Lami's Theorem – (Only applicable for equilibrium of 3 concurrent forces)
- ***II.*** Method of resolution – (applicable to equilibrium of any no. of forces)
- ***III.*** Method of moment.

- ***Condition of equilibrium :***

a. A body is said to be equilibrium under the action of concurrent forces if, **ΣH = 0 & ΣV = 0.**

b. A body is said to be equilibrium under the action of coplanar forces if, ***Σ M = 0***

In general, if a body is equilibrium under the action of system of forces then it must satisfy these 3 conditions, i.e. **ΣH = 0 & ΣV = 0 & *Σ M = 0.***

- **LAMI'S THEOREM :-**

It states that, *"If three forces acting at a point are in equilibrium, each force is proportional to the sine of angle between the other two."*

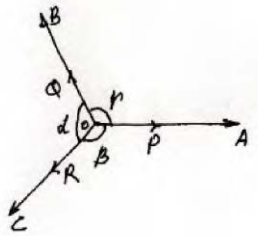
Let 3 forces P, Q & R acting at point 'O' along the lines OA, OB, OC respectively.

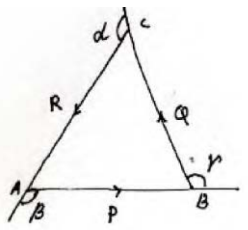
Let ∠ BOC = α , ∠ AOC = β & ∠ AOB = γ

Construct a triangle, whose sides are parallel to OA, OB & OC. Produce CA to D & we see that ∠ BAD = γ

Then, ∠ BAC = 180^o - γ

Similarly, ∠ BCA = 180^o - β & ∠ CBA = 180^o - α





$$\frac{AC}{\sin(\pi-\alpha)} = \frac{AB}{\sin(\pi-\beta)} = \frac{BC}{\sin(\pi-\gamma)}$$

or $\frac{AC}{\sin\alpha} = \frac{AB}{\sin\beta} = \frac{BC}{\sin\gamma}$ i.e. $\frac{P}{Sin\alpha} = \frac{Q}{Sin\beta} = \frac{R}{Sin\gamma}$

- **VARIOUS TYPES OF SUPPORTS WITH THEIR REACTIONS -**

A beam may have the following types of supports with reactions –

a. Beam with simply supported ends can carry only vertical loads & the reactions of the supports are all vertical.

b. Beam with one end hinged & the other end freely supported on rollers –

The reactions at the roller end is always at right angles to plane on which the rollers are placed.

The reaction at the hinged (or pinned) end may be either vertical or inclined. If a beam with one end hinged & the other end on roller supports, carries only vertical loads, the reactions at both the supports will be vertical. But if such a beam carries inclined loads, the reaction on roller support will be vertical (if the supporting plane of roller is horizontal) or at right angles to the supporting plane (when the supporting plane of the roller is inclined) & the reaction at the hinged end will be inclined. In other words, in such a case, the hinged end will carry the horizontal component as well as vertical components of loads.

c. In case of beam having both ends fixed, the reactions at the fixed ends are assumed to be parallel to the loads. In case of inclined loads, the horizontal thrust is assumed to be equally shared by supports.

1. ***OBJECTIVE QUESTIONS*** :
   a. ***Define force & state its units ?**
   b. ***Define resultant of a system of forces ?**
   c. **Differentiate between components & resolved parts of a force ?**
   d. **State polygon law of forces ?**
   e. **State triangle law of forces ?**
2. ***SHORT QUESTIONS*** :
   a. **State & prove parallelogram law of forces ?**
   b. ***State & prove Lami's theorem ?**
   c. **A particle is arced on by three forces 2, 2$\sqrt{2}$ & 1 kN. The first force is horizontal & towards the right, the second acts at 45° to the horizontal & inclined right upward & the third is vertical. Determine the resultant of the given forces.**
   d. **Determine analytically the resultant of two concurrent forces of 100 kN & 200 kN acting at an angle of 60°.**
   e. **Prove that the algebraic sum of resolved parts of two concurrent parts along any direction is equal to the resolved part of their resultant along the same direction .**
   f. **What is the difference between components & resolved parts of a force ?**
   g. **State the conditions of equilibrium of forces ?**
   h. **Explain the analytical determination of resultant of number of concurrent forces ?**
   i. **The forces 20 N, 30 N, 40 N, 50 N, 60 N are acting at one point of a regular hexagon towards the other five angular points taken in order. Find the magnitude & direction of the resultant force ?**
3. ***LONG QUESTIONS*** :
   a. **Forces 4, P, Q, 8 & 6 N act respectively along AB, CA, AD, AE & FA of regular hexagon ABCDEF & are in equilibrium. Find P & Q ?**
   b. **Four horizontal wires are attached to a vertical telegraph post & they exert the following pulls on the post – 20 kN due to North, 40 kN due to South West, 30 kN due to East, 50 kN due to South East. Find the magnitude & direction of the resultant pull ?**
   c. **A roller of weight 200 N rests on a smooth horizontal plane & is connected by an ring AC as shown in figure. If there is a horizontal force P of 100 n acting at C . Find the tension on the string AC & the reaction at B. Assume angle $\angle$BAC = 30°.**
   d. **A spherical ball of weight 100 kgf rests on two smooth inclined planes whose sides are inclined at 30° & 60° to the horizontal. Find the reaction of each inclined plane on the ball ?**
   e. **Two identical roller each weights 100 N are supported by an inclined plane & a vertical wall as shown in fig. Assuming smooth surface, find the reactions induced at the point A, B, C & D ?**

F R I C T I O N

INTRODUCTION :

When an external force is applied to a body to move it over the surface of another body, an opposing force comes into play along the common surface of contact of two bodies. This opposing force is called '*friction*' or '*frictional resistance*' or '*frictional force*'.

In other words, Whenever the surfaces of two bodies are in contact there will be a limited amount of resistance to sliding between them, which is called '*friction*' or '*force of friction*'.

- **UNIT :** Since, friction is nothing but a kind of force, its unit will be the same as that of force. Hence unit of friction will be N, kN etc..
- **TYPES** : Friction is of two types-

a. **Static friction** : (Between non-moving surfaces) - It is the friction experienced by a body when it is at rest. (It is the friction when the body tends to move)

b. **Dynamic friction** : (Between moving surfaces) - It is the friction experienced by a body when it is in motion. It is also called as kinetic friction. It is of two types –

I. **Sliding friction** : It is the friction, experienced by a body when it slides over another body.

II. **Rolling friction** : It is the friction experienced by a body when it rolls over another body.

- **FLUID FRICTION :** When a body moves through a fluid (i.e. a liquid or a gas), it experiences an opposing force. This opposing force offered by the fluid is called '*fluid friction*'.
- **CAUSE OF FRICTION :** Friction is mainly due to roughness of surfaces of the bodies in contact.
- **LAWS OF FRICTION : (PROF. COULOMB in 1871)**

1. The total friction that can be developed is independent of magnitude of the area of contact.

2. The total friction that can be developed is directly proportional to normal reaction (i.e. $F \propto R$).

3. For low velocities of sliding, the total friction that can be developed is practically independent of the velocity, although the experiments show that the force 'F' necessary to start sliding is greater than that necessary to maintain sliding.

- **LAWS OF STATIC FRICTION :**

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.

2. The force of friction is exactly equal to impressed force (the resolved component of the applied force, along the plane surface).

3. The magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces. i.e. F / R = a constant .

4. The force of friction is independent of area of contact between the two surfaces.

5. The force of friction depends upon the roughness of the surfaces & upon the materials of which the bodies in contact are made.

- **LAWS OF DYNAMIC FRICTION :**

1. The force of friction (F) always acts in a direction opposite to that in which the body is moving.

2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between two surfaces i.e. F / R = a constant. But, this ratio is slightly less than that in case of limiting friction.

3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

- **LIMITING FRICTION (F) :** The maximum value of friction force, which comes into play, when a body just begins to slide over the surface of the other body, is known as 'limiting friction'.

- When the applied force is less than the limiting friction, the body remains at rest & the friction is called 'static friction'.

- When the applied force is more than limiting friction, the body begins to move in the direction of applied force.

- Force of friction is always parallel to the plane surface & is equal & opposite to the impending force.

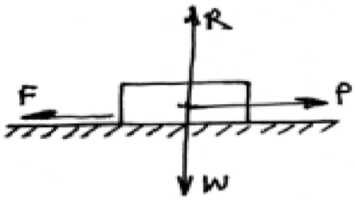
- **NORMAL REACTION (R) :**

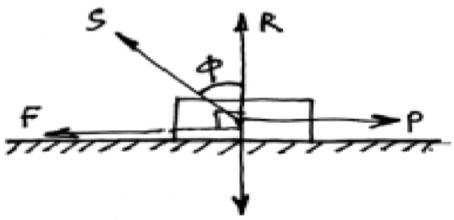
Whenever a body lying on a plane, is in equilibrium, its weight acts vertically downwards. The surface in turn, exerts an upward reaction on the body. This reaction is always perpendicular to the plane surface is called 'normal reaction '. It is denoted by ' R ' or R_N.

- The force of friction (F) is directly proportional to Normal reaction (R)

i.e. $F \propto R$ or $\mathbf{F = \mu . R}$

Where, μ = Co-efficient of friction





- **CO-EFFICIENT OF FRICTION (μ):**

From the law of static friction, we know that limiting friction is directly proportional to the normal reaction between two bodies in contact.

$\therefore$ $F \propto R$ Where, F = Limiting friction &

R = Normal reaction between two bodies in contact.

i.e. $F = \mu.R$ Where, μ = Co-efficient of friction.

$\therefore$ $\boldsymbol{\mu = F / R}$

Hence, Co-efficient of friction is the ratio of the limiting friction to the normal reaction between two bodies in contact.

- **RESULTANT REACTION or TOTAL REACTION (S) :**

S is the resultant of F & R

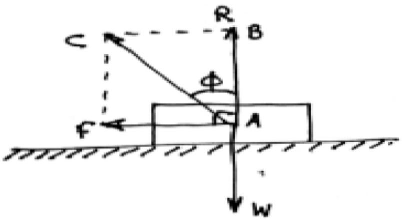
Where, F = Limiting friction

& R = Normal reaction.

As the included angle between F & R is 90˚

$\therefore$ The resultant, $S = \sqrt{R^2 + F^2}$

$\therefore$ **Total reaction , $S = \sqrt{R^2 + F^2}$**



- **ANGLE OF FRICTION (φ) :**

Angle of limiting friction may be defined as the angle between the resultant reaction (S, the resultant of F & R) & the normal to the plane (R) on which the motion of the body is impending.

$\boldsymbol{\mu = F / R = \text{Tan}\ \varphi}$

Where, φ = Angle included between total reaction & normal reaction = Angle of friction.

- **RELATION BETWEEN ANGLE OF FRICTION & CO-EFFICIENT OF FRICTION :**

F = Limiting friction

R = Normal reaction

φ = Angle of friction

From the geometry of fig.

In triangle ABC , $\tan \varphi = BC / AB = F / R$

But we know that, $F / R = \mu$

Where, μ = Co-efficient of friction.

$\therefore$ **Tan φ = μ**

Hence, Tan of the angle of friction is the co-efficient of friction.

- **ANGLE OF REPOSE (α) :**

The inclination of the plane on which the body, free from external forces can start sliding down is called the angle of repose.

- **RELATION BETWEEN ANGLE OF FRICTION (φ) & ANGLE OF REPOSE (α) :**

In an inclined plane,

Let, W = Weight of the body

α = Angle of repose for the body

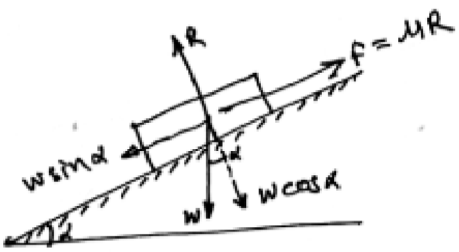
F = Limiting friction

R = Normal reaction

Now resolved part of W along a direction perpendicular to the inclined plane = W cos α & resolved part of W along the inclined plane

=W sin α

Considering equilibrium of body on inclined plane, we get



I. Algebraic sum of ball resolved parts of forces along inclined plane = 0 i.e. F - W sin α = **0** or F = W sin α

μ R = W sin α ………….. **(1)** **(** as F = μR)

II. Algebraic sum of all resolved parts of forces along a direction perpendicular to the inclined plane = 0.

R - W cos α = 0 or R = W cos α . . . . . . **(2)**

By putting the value of 'R' in equation (1), we get

μ .W cos α = W sin α

or $\frac{W\, sin\, \alpha}{W\, cos \alpha}$ **= μ** or tan α **= μ**

But μ = Tan φ Where, φ = Angle of friction.

∴ Tan φ = Tan α or $\varphi = \alpha$

- **GENERAL METHOD OF FINDING - The force required to move a body along any plane (hor. & inclined)**

Force required to move a body along any plane means that the value of force for which the body will be just in equilibrium.

Under such conditions, the frictional force developed along the common surface of contact of two bodies is the ' limiting friction '.

a) Resolve all the forces acting on the body along the given plane & perpendicular to the plane.

b) Equate to 0 (zero) the algebraic sum of the resolved parts of forces along the plane.

c) Equate to 0 (zero) the algebraic sum of the resolved parts of forces perpendicular to the plane.

- **Horizontal pull/push required to move a body along a horizontal plane :**

P = μ . W

Where, μ = $Tan\ \varphi$ = co-efficient of friction

φ = Angle of friction

W = Weight of the body

P = Horizontal effort required to move the body along the plane.

- **Inclined pull required to move the body along a horizontal plane :**

P = $\frac{W sin\varphi}{cos(\alpha - \varphi)}$

Where, φ = Angle of friction

α = Angle made by line of action of the pull (P) with horizontal

W = Weight of the body

P = Inclined pull required to move the body along the plane.

- **Least inclined pull required to move a body along a horizontal plane :**

We know that, the inclined pull required to move a body alonga horizontal plane is **P =** $\frac{W sin\varphi}{cos(\alpha - \varphi)}$

P will be least when $cos(\alpha - \varphi)$ Is maximum i.e. $cos(\alpha - \varphi) = 1$ i.e. $(\alpha - \varphi) = 0$ i.e. $\alpha = \varphi$

$P_{least} = \frac{W sin\varphi}{1} = W sin\varphi$

- **Inclined push required to move a body along a horizontal plane :**

P = $\frac{W sin\varphi}{\cos(\alpha + \varphi)}$

Least inclined push required to move a body along a horizontal plane :

P_{least} **= Wsinφ**

- **EQUILIBRIUM OF A BODY LYING ON A ROUGH HORIZONTAL PLANE :**

A body lying on a rough horizontal plane is always equilibrium. But whenever a force is applied to it, the body will tend to move in the direction of applied force. In such cases, equilibrium of body is studied by -

I. Resolving the forces horizontally

II. Resolving the forces vertically.

The values of force of friction is obtained from the relation : ***F = μR***

- **EQUILIBRIUM OF A BODY LYING ON A ROUGH INCLINED PLANE :**

Consider a body, of weight 'W' lying on a rough plane inclined at an angle $'\alpha'$ with the horizontal.

If the inclination of the plane, with the horizontal, is less than the angle of friction (i.e $\alpha < \varphi$), the body will be automatically in equilibrium. In this condition, if the body is required to moved upward or downward, a corresponding force is required.

If the inclination of the plane is more than the angle of friction, the body will move down, & an upward force will be required to resist the body from moving down.

For movement of the body, following forces are important –

- ***a.*** Force acting along the inclined plane.
- ***b.*** Force acting horizontally.
- ***c.*** Force acting at some angle with the inclined plane.

➢ **EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE, SUBJECTED TO A FORCE ALONG THE INCLINED PLANE :**

Here, α = Angle of inclination & φ = Angle of friction.

a. Minimum force (P_1) necessary to keep the body in equilibrium, when it is **at the point of sliding downwards :**

Resolving the forces parallel to the plane surface,

$P_1 + F = W\sin\alpha$

$P_1 + \mu R = W\sin\alpha$ .... (1) (as $F = \mu R$)

Resolving the forces perpendicular to the plane,

$R = W\cos\alpha$ .... (2)

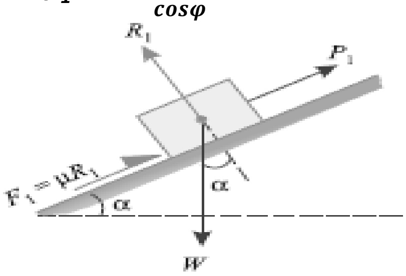
By putting the value of R in equation (1), we get

$P_1 + \mu W\cos\alpha = W\sin\alpha$

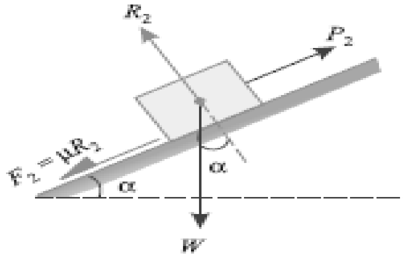
$P_1 = W\sin\alpha - \mu W\cos\alpha = W\sin\alpha - \tan\varphi . W\cos\alpha$ (as $\mu = \tan\varphi$)

$= W\sin\alpha - W\frac{\sin\varphi}{\cos\varphi}.\cos\alpha = W\sin\alpha.\cos\varphi - W.\sin\varphi.\cos\alpha$

$$P_1 = \frac{W.\sin(\alpha - \varphi)}{\cos\varphi}$$



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

b. Maximum force (P_2) necessary to keep the body in equilibrium, when it is **at the point of sliding upwards :**

Resolving the forces, parallel to the plane –

$P_2 = F + W\sin\alpha$ i.e. $P_2 = \mu.R + W\sin\alpha$ .... (1)

Resolving the forces, perpendicular to the plane -

$R = W\cos\alpha$ .... (2)

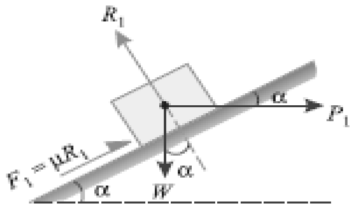
By putting the value of R in equation (1), we get –

$P_2 = \mu.W\cos\alpha + W\sin\alpha = \tan\varphi\, W\cos\alpha + W\sin\alpha = \frac{\sin\varphi.W\cos\alpha}{\cos\varphi} + W\sin\alpha$

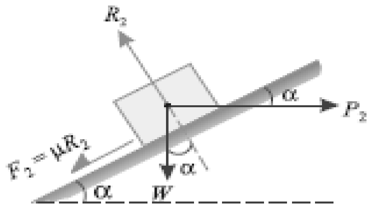
$= \frac{W(\sin\varphi.\cos\alpha + \sin\alpha.\cos\varphi)}{\cos\varphi}$

$$P_2 = \frac{W\sin(\alpha + \varphi)}{\cos\varphi}$$

➢ **EQUILIBRIUM OF A BODY ON A INCLINED PLANE, SUBJECTED TO A FORCE ACTING HORIZONTALLY :**



(a) Body at the point of sliding downwards



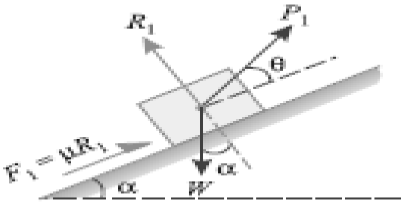
(b) Body at the point of sliding upwards

a. **Minimum force (P_1) = W tan($\alpha - \varphi$) when, $\alpha > \varphi$**
= Wtan ($\varphi - \alpha$) when $\varphi > \alpha$

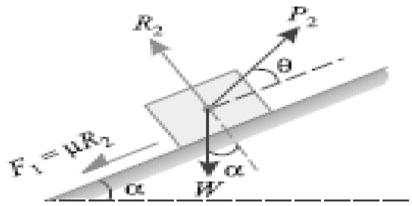
b. **Maximum force (P_2) = W tan ($\alpha + \varphi$)**

- **EQUILIBRIUM OF A BODY ON INCLINED PLANE, SUBJECTED TO A FORCE ACTING AT SOME ANGLE WITH PLANE :**

Here, α = Angle of inclination
θ = Angle of required force to the horizontal
φ = Angle of friction



(*a*) Body at the point of sliding downwards



(*b*) Body at the point of sliding upwards

a. **Minimum force (P_1) = $\dfrac{W\sin(\alpha - \varphi)}{\cos(\theta + \varphi)}$**

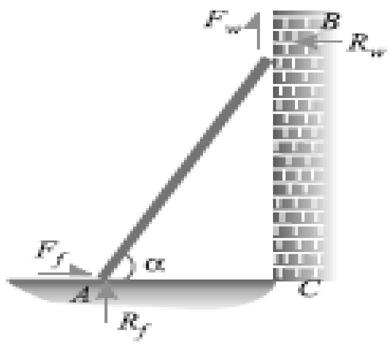
b. **Maximum force (P_2) = $\dfrac{W\sin(\alpha + \varphi)}{\cos(\theta - \varphi)}$**

- **LADDER FRICTION :**

The ladder is a device for climbing on the walls. It consist of two long uprights of wood, iron or rope connected by a no. of cross pieces called rungs. These rungs serve as steps.

Equilibrium of a ladder :

Consider a ladder 'AB' rests with one end 'B' against a vertical wall 'BC' & the other end 'A' on a horizontal plane 'AC'. When the ladder is on the point of slipping, its end 'A' has a tendency to move towards the left & the end 'B' has a tendency to move downward. Hence, the limiting friction acting on ladder at 'A' i.e. F_A will act towards the right (i.e in the direction AC) and the limiting friction acting on ladder at 'B' i.e. F_B will act upward (i.e. in the direction CB).

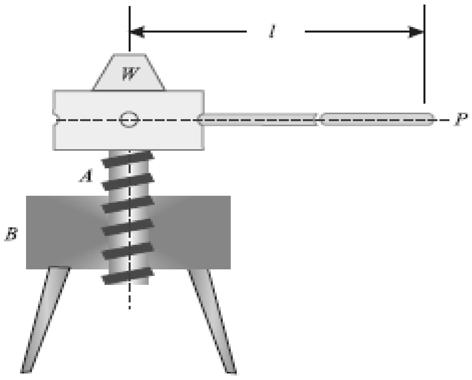


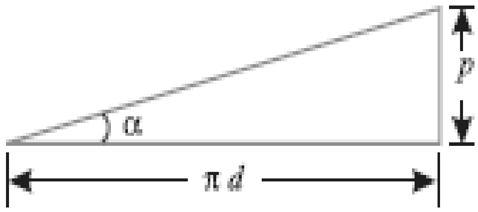
The ladder is under the action of co-planer non-concurrent forces, hence the condition of equilibrium for ladder are –

a. $\sum H = 0$,
b. $\sum V = 0$ &
c. $\sum M = 0$.

Where , $\sum$ **H** = Algebraic sum of resolved parts of forces along the horizontal plane.
$\sum$ **V** = Algebraic sum of resolved parts of forces along the vertical direction.
$\sum M$ = Algebraic sum of moments of forces about any point in their plane (on ladder).

➢ SCREW JACK :





A screw jack is a device for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.

A screw jack consists of a threaded rod, called screw rod or simply screw. The screw has square threads, on its outer surface, which fit into the inner threads of the jack. The load to be raised or lowered, is placed on the head of the screw, which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

In one complete turn of a screw thread, be imagined to be unwound, from the body of the screw & developed, it will form an inclined plane.

Let, p = pitch of the screw,
d = Mean diameter of the screw,
r = mean radius of the screw, & α = Helix angle.
From the geometry of figure, tan $\alpha = \frac{p}{\pi \mathrm{d}}$

Let, P = Effort applied at the mean radius of the screw jack to lift the load,
W = Weight of the body to be lifted,
μ = Co-efficient of friction, between the screw & nut.
Let φ = Angle of friction, such that μ = tan φ

The force applied on the lever of screw jack is considered to be horizontal, then effort required to lift the weight - **P = W tan ($\alpha + \varphi$)**

The effort required to lowered the weight - **P = W tan($\alpha - \varphi$) when, $\alpha > \varphi$**
(at mean radius) **= Wtan ($\varphi - \alpha$) when $\varphi > \alpha$**

The effort required at the end of the lever length 'l', **$P_1 = \frac{P \cdot r}{l}$**

Where, r = mean radius = $\frac{(Outer\ radius + inner\ radius\ of\ screw)}{2}$

l = length of lever.

➢ EFFICIENCY OF A SCREW JACK :

The effort (P) required at the mean radius of a screw jack to lift the load (W),

P = W tan ($\alpha + \varphi$) …….. (1)

Where, α = Helix angle, &
μ = Co-efficient of friction, between the screw & nut
= tan φ where φ = angle of friction

If there would have been no friction between the screw & the nut, then $'\varphi'$ will be zero. In such a case, the value of effort ($P0$) necessary to raise the same load, will be given by –

P_0 = W tan α (substituting φ = 0 in equation (1))

∴ Efficiency, $\eta = \frac{Ideal\ effort}{Actual\ effort} = \frac{P0}{P} = \frac{W \tan \alpha}{W \tan(\alpha + \varphi)} = \frac{\tan \alpha}{\tan(\alpha + \varphi)}$

It shows that, the efficiency is independent of the weight lifted or effort applied.
The efficiency of the screw jack may also be written as –

$$M.A = W/P_1 = \frac{W}{\frac{P \cdot r}{l}} = \frac{W \cdot 2l}{P \cdot d} = \frac{W \cdot 2l}{W \tan(\alpha + \varphi) \cdot d} = \frac{2l}{\tan(\alpha + \varphi) \cdot d}$$

$$V.R. = \frac{Distance\ moved\ by\ effort\ (P) in\ one\ revolution}{Distance\ moved\ by\ load\ (W) in\ one\ revolution} = \frac{2\pi l}{p} = \frac{2\pi l}{\tan\alpha \cdot \pi d} = \frac{2l}{\tan\alpha \cdot d} \quad (as\ \tan\alpha = \frac{p}{\pi d}\ \&\ p = \tan\alpha \cdot \pi d)$$

∴ ***Efficiency,*** $\boldsymbol{\eta} = \frac{M.A}{V.R} = \frac{\frac{2l}{\tan(\alpha+\varphi).d}}{\frac{2l}{\tan\alpha .d}} = \frac{\tan\alpha}{\tan(\alpha+\varphi)}$

The ***efficiency to be maximum***, the value of sin ($2\alpha + \varphi$) should be greatest.

i.e. $2\alpha + \varphi = 90°$ or $\boldsymbol{\alpha = 45° - \frac{\varphi}{2}}$

➢ ***USEFUL & HARMFUL EFFECTS OF FRICTION :***

- ***Useful effects of friction are –***
  - ***a.*** When we write something on the blackboard or anywhere else, it is due to friction that the writing agent (i.e. chalk stick, pen, pencil etc.) doesn't slip from the grip of our hand.
  - ***b.*** It would be impossible for us to walk on the ground if there were no friction. If the frictional resistance is absent, the force exerted by us in the forward direction takes our feet forward & we will fall flat on the ground on our back.
  - ***c.*** A nail stuck into a piece of wood does not readily come out when pulled outward. It is the friction which offers resistance to the force exerted on the nail to pull it out.
  - ***d.*** When a ladder is placed with its one end against a vertical wall & the other end on a horizontal surface, it is the frictional resistances F_1 & F_2 of the wall & the horizontal surface respectively which prevent the ladder from slipping.
  - ***e.*** It is due to friction of the railway lines on the wheels acting in a direction opposite to the direction of the wheels that the train can advance in a linear direction. If the friction were absent, the wheels would have rotated about their respective axis remaining in the same position. The result is that the train wouldn't move. Friction plays the same role in moving motor cars & other vehicles on the road.
- ***Harmful effects of friction are*** –
  - ***a.*** Large amount of power is lost in friction in the bearings.
  - ***b.*** Large amount of power lost in friction in different parts of a heat engine (steam engine, I.C. engine etc.) so that power available at the engine crank shaft is less than power developed within the engine cylinder.
  - ***c.*** It is due to friction that the cutting tools used in various machine tools soon get worn out.
  - ***d.*** It is due to fluid friction between atmospheric air & outer surface of an aircraft that the body of the aircraft becomes heated. This heat is conducted into the passenger cabin & is to be removed by suitable air-conditioning arrangement.

CENTRE OF GRAVITY & MOMENT OF INERTIA

➢ CENTRE OF GRAVITY :

Centre of gravity of a body is defined as the point through which the resultant of all the parallel forces of attraction formed by the weight of the body is assumed to pass.

- ➢ Every particle of a body is attracted by the earth towards its centre with a force proportional to its mass. Those forces form a system of parallel forces & the centre of the system of forces is called the *centre of gravity* of body.
- ➢ ***Centroid*** : It is defined as a point where the whole area of the body is assumed to be concentrated. When the thickness of a body is not considered, then C.G. & centroid of the body are same.

The plane figure (known as lamina) like rectangle, triangle, circle etc. have area only but no mass. Hence, C.G. & centroid of such figures are at the same point.

- ***Method of finding C.G. of simple figures :***

Centre of gravity may be found out by following methods –

1. Geometrical consideration,
2. Graphical method, &
3. Moments.

- ***Centre of Gravity by method of Integration : (First principle)***

Consider an infinite small element of area dA at a distance of x & y from y-axis & x-axis.
Let x_c & y_c be the co-ordinates of centroid of a given section of area 'A'.
According to the principle of moment,

$y_c . \int dA = \int y\, dA$

Or $\quad y_c = \frac{\int y\, dA}{\int dA}$

Similarly, $x_c = \frac{\int x\, dA}{\int dA}$

- ***Centre of gravity of a uniform rod : (By Geometric Method)***

(A uniform rod means a rod which has the same diameter anywhere in its length & which is made up of the same material, so that weight of any particle of equal volume is the same anywhere in it.)

Let AB be a thin uniform rod, & 'G' be its middle point.
Since the rod is uniform, for any particle to the left of G there must be a similar particle of equal weight at equal distance to the right of 'G'.

Let w = weight of each particle acting at points P & Q such that PG = QG.

The weights of P & Q form two like parallel forces, each of magnitude 'w'. Hence their resultant = w + w = 2w, & this resultant evidently passes through 'G'. That is the centre of gravity of these two equal particles at P & Q is at 'G'.

Similarly, if we consider two other similar particles equidistant from G, we will find that the centre of gravity of these two particles also lie at 'G'. Since the entire rod may be considered to be made up of a no. of such pairs of identical particles equidistant from 'G', & since all these pairs of particles will have their centre of gravity at 'G', we can conclude that the centre of gravity of the whole rod AB is at 'G'.

Alternate method : (By Integration Method)

Let *l* = length of the rod whose weight per unit length is the same throughout, because the rod is uniform.

Also, *let w* = weight per unit length of the rod. Then, total weight of the rod = *w.l* which acts through 'G' which is the centre of gravity of the rod.

Let X = distance of 'G' from the left end O of the rod.

At any distance 'x' from O , let us consider an elementary length of very small magnitude *dx as in fig..*
Elementary weight of elementary length = *w.dx*

Moment of this elementary weight about O = *dx.x*

The rod may be considered to be made up of a number of such elementary weights. The algebraic sum of the moments of all such elementary weights about O = $\int_0^l w.dx.x$ = w $\int_0^l x.dx$

$= w\,[x^2/2]^l_0 = wl^2/2$

We know that the algebraic sum of the moments of any number of forces about any point in their plane is equal to the moment of their resultant about the same point.

Hence, $\quad wl^2/2 = wl.X$ or $X = l/2$

Hence *centre of gravity of a uniform rod lies at its middle point.*

- ***Centre of Gravity of a Uniform Triangular Lamina : (By Geometric Method)***

  Let D be the middle point of the base BC of a uniform triangular lamina ABC.
  Also let XY be any thin strip parallel to the base BC intersecting the median AD at O.
  The triangles AXO & ABD are equiangular $\therefore \frac{XO}{BD} = \frac{AO}{AD}$ ..... (1)
  Again triangles AOY & ACD are equiangular, $\therefore \frac{YO}{CD} = \frac{AO}{AD}$ .... (2)
  $\therefore$ eq. (1) = eq. (2) i.e. $\frac{XO}{BD} = \frac{YO}{CD}$

But BD = CD $\therefore$ XO = YO. Hence O is the middle point of the strip XY, i.e. O is the C.G. of the strip XY. In other words, C.G. of the strip XY lies on the median AD.
Similarly it can be shown that the C.G. of any other strip parallel to BC will lie on the median AD. Since the whole triangular lamina ABC may be considered to be made up of a no. of such thin strips parallel to BC, we can conclude that , the C.G. of the whole area ABC will lie on the median AD.

By taking thin strips parallel to AC, similarly it can be shown that the C.G. of the whole triangular ABC will also lie on the median BE. Hence C.G. of a uniform triangular lamina lies at the point of intersection of the medians of the triangles.

Let G be the C.G. of the Δ ABC. Point D is joined with point E.
Evidently, Δs GDE & ABC are equiangular,

$\therefore \frac{GD}{AG} = \frac{DE}{AB} = \frac{\frac{1}{2}AB}{AB} = ½$ $\therefore$ 2 GD = AG.

Adding GD to both sides , we get - 2GD + GD = AG + GD or 3GD = AD or **GD = 1/3 AD**
It can be shown that the vertical the vertical height of G from the base BC is also equal to
1/3 x vertical height of A from BC (1/3 h).

- ***C.G. OF A UNIFORM RECTANGULAR LAMINA : (By Geometric method)***

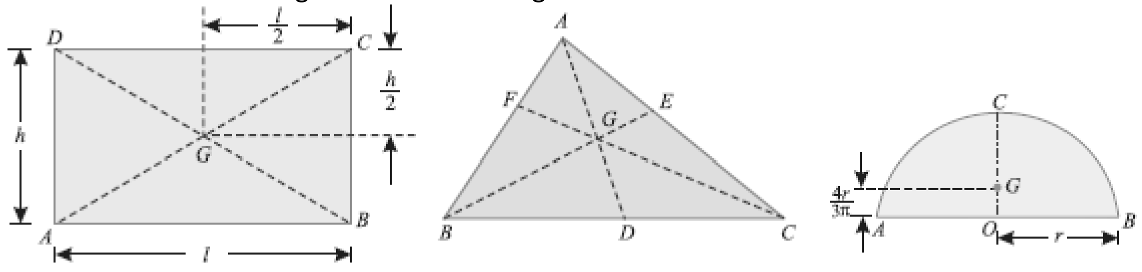
Let ABCD be a uniform lamina in the form of a rectangle. Let E, F, L & M be the middle points of the sides AB, CD, AD & BC respectively. The point E is joined with point F, & point L is joined with point M.

Let us consider any thin strip XY parallel to AB intersecting EF at P. Evidently, P is the middle point of XY. Hence P is the C.G. of the strip XY i.e., C.G. of the strip XY lies on the line EF.

Similarly, taking any other strip parallel to AB, it can be shown that the C.G. of that strip also will lie on the line EF. Now, the whole rectangle ABCD may be considered to be made up of a number of thin strips parallel to AB, and since all those strips will have their C.G. lying on EF. It can be concluded that the C.G. of the whole rectangle ABCD will lie on the line EF.

Similarly, taking strips parallel to BC (like GH & so on …), we can show that the C.G. of the whole rectangle ABCD will lie on the line LM.

Hence, C.G. of the whole rectangle lies at the intersection of the lines EF & LM which is also the point of intersection of the diagonals of the rectangle.



Rectangle Triangle Semicircle

- ***C.G. OF A CIRCULAR LAMINA : (By Geometric Method)***

Let AB be any diameter of a uniform circular lamina having O as centre.
Any thin strip XY taken at right angles to AB intersecting it at G_1. G_1 is the middle point of XY.
Therefore, G_1 is the C.G. of the strip XY. C.G. of the strip XY lies on the diameter AB.

Similarly, it can be shown that the C.G. of any other strip taken at right angles to AB will lie on the diameter AB. Since the whole circular lamina can be considered to be made up of a number of such strips at right angles to the diameter AB, & since all those strips will have their C.G. of the whole circular lamina will also lie on the diameter LM.

Hence, C.G. of the whole circular lamina will be the point of intersection of the diameters of the circular lamina. C.G. of the whole circular lamina will be the centre of the circle. $x_c = y_c = R$

- ***C.G. OF A SEMI-CIRCULAR LAMINA :***

Consider an elemental area OCD of angle '$d\theta$' as shown in fig. When '$d\theta$' is small, CD = R $d\theta$ can be considered as a line & OCD as a triangle, then radial distance of centroid of OCD from 'O' is 2/3 R.

$x = 2/3\, R \cos\theta$ & $y = 2/3\, R \sin\theta$

Area of the elemental strip, $dA = ½\, R\,(R\, d\theta) = ½\, R^2\, d\theta$

Total area of the semicircle, $A = \int dA = \int_0^{\pi} (R^2/2)\, d\theta = \pi R^2/2$

$\int y dA = \int_0^{\pi} ½\ R2\ d\theta . 2/3\, R \sin\theta = 1/3\, R^3 \int_0^{\pi} \sin\theta d\theta = 1/3\, R^3\, (-\cos\theta)$

$= 1/3\, R^3 . 2 = 2/3 R^3$

$$y_c = \frac{\int y dA}{\int dA} = (2/3R^3)/(\pi R^2/2) = 4R/3\pi \quad \& \quad x_c = R$$

- ***CENTRE OF GRAVITY OF PLANE FIGURES :***

a. ***Straight line*** : The C.G. of a straight line is at its mid point.

b. ***Triangle :*** The C.G. of a triangle is at a point where the three medians intersect each other.

c. ***Rectangle :*** The C.G. of a rectangle (or parallelogram) is at point of intersection of diagonals, i.e. at the midpoint of its length as well as breadth.

d. ***Semi-circular lamina :*** The C.G. of a semicircular lamina is at a distance ***4R/3π*** from its base, measured along vertical radius.

e. ***Trapezium :*** The centre of gravity of a trapezium with parallel sides a & b is at a distance of $\frac{h}{3} \times \left(\frac{b+2a}{b+a}\right)$ measured from the side b.

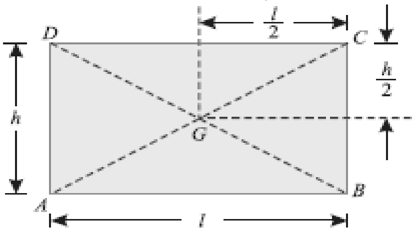
f. ***Circular sector :*** The centre of gravity of a circular sector making semi-vertical angle α is at a distance of $\frac{2r}{3}\frac{\sin\alpha}{\alpha}$ from the centre of the sector measured along the central axis.

g. ***Hemisphere :*** The C.G. of the hemisphere is at a distance of 3R/8 from its base, measured along vertical radius.

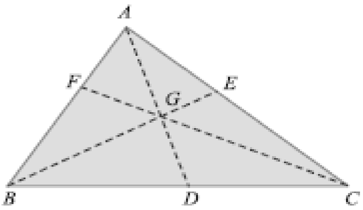
h. ***Right circular solid cone :*** The C.G. of cone is at a distance of h/4 from the base, measured along the vertical axis.

i. ***Cube :*** The C.G. of a cube is at a distance of l/2 from every face (where l is the length of each side).

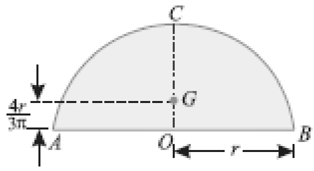
j. ***Sphere :*** The C.G. of a sphere is at a distance of d/2 from every point (where d is the dia. of sphere).



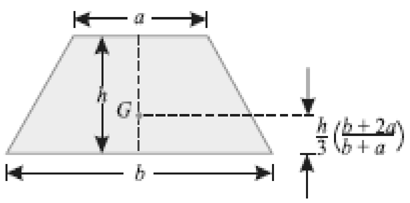
Rectangle



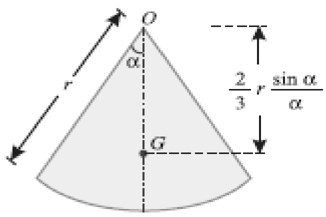
Triangle



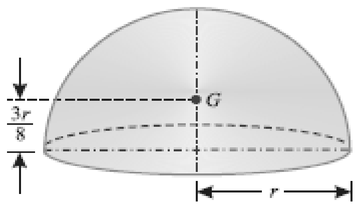
Semicircle



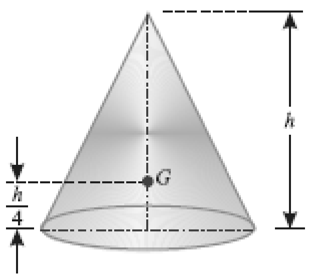
Trapezium



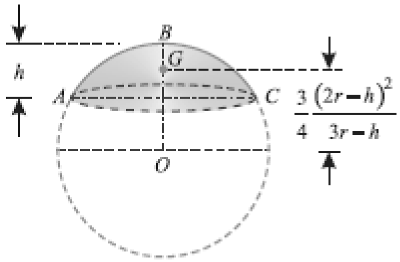
Circular sector



Hemisphere



Right circular solid cone



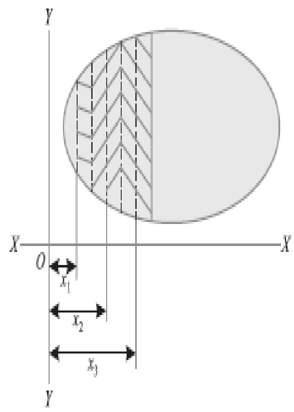
Segment of a sphere

- ***CENTRE OF GRAVITY BY MOMENTS :***

The centre of gravity of a body may also be found out by moments as discussed below –

Consider a body of mass M whose C.G. is required to be found out. Divide the body into small masses, whose centre of gravity are known.

Let m_1, m_2, m_3……….. etc. be the masses of the particles & (x_1, y_1), (x_2, y_2) & (x_3, y_3)……. Be the co-ordinates of the centre of gravity from a fixed point 'O' as in fig.



Let x & y be the co-ordinates of the C.G. of the body. From the principle of moments, we know that, $Mx = m_1x_1 + m_2x_2 + m_3x_3$ …….

Or $x = \frac{\sum mx}{M}$

Similarly, $y = \frac{\sum my}{M}$ Where, $M = m_1 + m_2 + m_3 + \ldots$

- ***SYMMETRICAL & UNSYMMETRICAL SECTIONS :***

If a figure or section is divided into two parts by an axis & any part of the figure is the mirror image of the other part then the section is said to be symmetrical about that axis. Otherwise the section is un-symmetrical.

Centroid always lies on the axis of symmetry (If a section is symmetrical to both x-axis & y-axis ,then the Centroid is the intersection of two axis of symmetry.

- A rectangular or square section is symmetrical about both x-axis & y-axis.
- An isosceles triangle or an equilateral triangle is symmetrical about y-axis but un-symmetrical about x-axis. (If the base of the triangle is kept horizontal)
- A circular section is always symmetrical about both x-axis & y-axis.
- A right angled triangle is un-symmetrical about both x-axis & y-axis.
- A semi circle is symmetrical about y-axis but un-symmetrical about x-axis.
- A quadrant is un-symmetrical about both x-axis & y-axis.

Composite section : *When two or more regular or irregular sections are added, the combined section is called as composite section.*

  - *T-section* is symmetrical about y-axis, but un-symmetrical about x-axis.
  - *I-section* is symmetrical about both x-axis & y-axis.
  - An *angle section* (L-section) is un-symmetrical about both x-axis & y-axis.
  - A *channel section* (C-section) is symmetrical about x-axis & un-symmetrical about y-axis.

- ***AXIS OF REFERENCE :*** *The co-ordinates of centroid of a lamina is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference).*
  - The bottom most line of figure is assumed to be x-axis of reference & extreme left most line of the figure is assumed to be y-axis of reference, if the figure is not marked with any axis.
  - If a figure is symmetrical about y-axis, mark the x-axis of reference & locate y_c.
  - If a figure is symmetrical about x-axis, mark the y-axis of reference & locate x_c.
  - If a figure is un-symmetrical, then mark both the axis of reference & find x_c & y_c.
  - If a figure has already been marked with some given axes, then referring to that given axes, locate the co-ordinates of the centroid.

Where, x_c = x co-ordinate of centroid = Horizontal distance of centroid from y-axis.

y_c = y co-ordinate of centroid = Vertical distance of centroid from x-axis.

- ***CENTRE OF GRAVITY OF PLANE FIGURE :***

Let x_C & y_C be the co-ordinates of the C.G. with respect to some axis of reference, then

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots}{a_1 + a_2 + a_3}$$

and

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots}{a_1 + a_2 + a_3 + \ldots}$$

Where a_1, a_2, a_3 . . . . etc. are the areas into which the whole figure is divided.

x_1, x_2, x_3 . . . are the respective co-ordinates of the areas a_1, a_2, a_3 . . . . on X-X axis with respect to same axis of reference.

y_1, y_2, y_3 . . . are the respective co-ordinates of the areas a_1, a_2, a_3 . . . . on Y-Y axis with respect to same axis of reference.

While using above formula distances must be measured from the same axis of reference & on the same side of it. If the figure is on both sides of the axis of reference, then the distances on one direction are taken as positive & those in opposite direction must be taken as negative.

- ***CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS :***

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the C.G. of the body is very much simplified ; as we have only to calculate either ***x_c or y_c.*** This is due to the reason that the C.G. of the body will lie on the axis of symmetry.

- ***CENTRE OF GRAVITY OF UN-SYMMETRICAL SECTIONS :***

Sometimes the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of ***x_c*** & ***y_c***.

- ***CENTRE OF GRAVITY OF SECTIONS WITH CUT OUT SECTIONS :***

The C.G. of such a section can be found out by considering the whole area of the figure, first as a complete one, and then deducting the area of the cut out section from that.

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} \quad \text{and} \quad \bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

Where, a_1 is the area of whole figure. & a_2 is the area of cut out section.

CENTRE OF GRAVITY OF SOLID BODIES

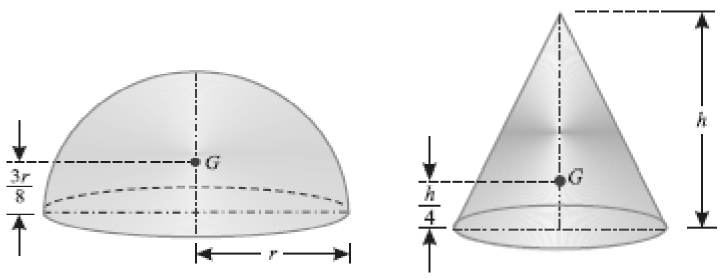
The centre of gravity of solid bodies (such as hemispheres, cylinders, right circular solid cones etc.) is found out in the same way as that of plane figures. The only difference, between the plane figures and solid bodies, is that in the case of solid bodies, we calculate volumes instead of areas. The volumes of few solid bodies are given below :

1. Volume of cylinder $= \pi \times r^2 \times h$
2. Volume of hemisphere $= \frac{2\pi}{3} \times r^3$
3. Volume of right circular solid cone $= \frac{\pi}{3} \times r^2 \times h$

where r = Radius of the body, and

h = Height of the body.

Note. Sometimes the densities of the two solids are different. In such a case, we calculate the weights instead of volumes and the centre of gravity of the body is found out as usual.



Hemisphere   Right circular solid cone

➢ MOMENT OF INERTIA :

The moment of a force about a point is the product of the force (P) & perpendicular distance (x) between the point & line of action of force. i.e. $M = P . x$

This moment is called *first moment of force*.

If this moment is again multiplied by perpendicular distance (x) between the point & line of action of force (i.e. $P.x.x = P.x^2$), then this quantity ($P.x^2$) is called *moment of moment of force* or *second moment of force* or *moment of inertia* (M.I.)

Sometimes, instead of force, mass or area of body or figure is taken into consideration, then second moment is known as *second moment of area* or *second moment of mass*.

- ***UNITS OF M.I. :*** The units of M.I. of a plane area depends upon the units of area & length.
  - ***a.*** If area in m^2 & length in m, then M.I. in m^4.
  - ***b.*** If area in cm^2 & length in m, then M.I. in cm^4. & so on …
- ***M.I. OF PLANE AREA :***

Consider a plane area, whose moment of inertia is to be determined.

Let's split up the whole area into a no. of small elements.

Let $a_1, a_2, a_3, a_4. . . . .$ be the area of small elements &

r_1, r_2, r_3, r_4 be the distance of elements from the line 1-1 about which M.I. is required to be found out.

M.I. of area, $I = a_1r_1^2 + a_2r_2^2 + a_3r_3^2 + a_4r_4^2$

$$I = \Sigma ar^2$$

- ***RADIUS OF GYRATION :***

If the entire mass of the body be assumed to be concentrated at certain point, at a distance 'K' from the given axes, such that, $M.K^2 = 1$

$\therefore K = \sqrt{I/M}$

Where, I = mass moment of inertia.

M = mass

K = the distance from axes, Radius of Gyration.

- ***THEOREM OF PARALLEL AXIS :***

It states that, *the moment of inertia of a plane section about an axis parallel to centroidal axis is equal to the moment of inertia of the section about the centroidal axis plus the product of the area of the section & the square of the distance between two axes.*

PROOF :

Let us consider a lamina whose M.I. about the axis A-A is to be determine.

Let A & G be the area & centroid of the lamina.

Let X-X & Y-Y be the centroidal axes at a distance of h_2 & h_1 from B-B & A-A respectively.

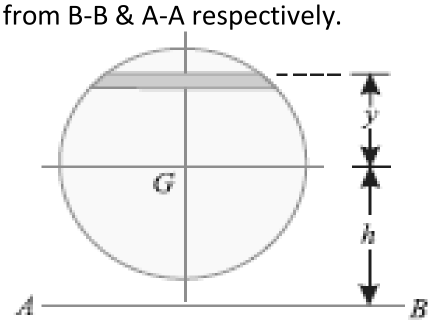
Consider an elemental area at a distance of 'x' from Y-Y & 'y' from X-X.

M.I. of elementary area about A-A $= a(h_1 + x)^2$

Now M.I. of whole area about the axis A-A,

$I_{AA} = \Sigma a (h_1 + x)^2$

$= \Sigma a (h_1^2 + 2h_1x + x^2) = \Sigma ah_1^2 + \Sigma 2ah_1x + \Sigma ax^2 = h_1^2 \Sigma a + 2h_1 \Sigma ax + \Sigma ax^2$



Theorem of parallel axis.

But, $h_1^2 \Sigma a = Ah_1^2$, where, A = total area of plane section.

Σax^2 = M.I. of whole area about Y-Y axis $= \mathbf{I_{G\,Y\text{-}Y}}$

$2h_1 \Sigma ax = 2h_1.0 = 0$

Here, $\mathbf{I_{AA} = I_{G\,Y\text{-}Y} + Ah_1^2}$

Similarly it can be proved that, $\boldsymbol{I_{BB} = I_{G\,X\text{-}X} + Ah_2^2}$

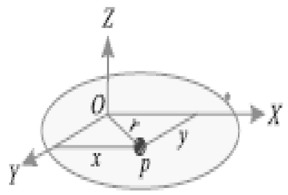
Where, $\boldsymbol{I_{G\,X\text{-}X}}$ = M.I. of section about X-X passing through centroid (G)

$\mathbf{I_{G\,Y\text{-}Y}}$ = M.I. of section about Y-Y passing through centroid (G).

- ***PERPENDICULAR AXIS THEOREM :***

It states that, *The M.I. of a lamina about an axis perpendicular to the plane lamina & passing through its C.G. is equal to the sum of the moment of inertia of the lamina about 2 mutually perpendicular axes passing through its C.G. & lying in the plane of lamina i.e.* $I_{ZZ} = I_{XX} + I_{YY}$.

Consider a lamina of area 'A'. Let 'XX' & 'YY' be the two mutually perpendicular axes in the plane of the lamina passing through its centroid 'G'.'ZZ' is the axis perpendicular to the plane of the lamina & is passing through the point of intersection of the axis 'XX' & 'YY'.



Theorem of perpendicular axis.

Consider an elementary area 'a'. Let x, y, z be the perpendicular distances of area 'a' from respective axis.

From the geometry, $z^2 = x^2 + y^2$

M.I. of plane area about y-axis = $\sum ax^2$

M.I. of plane area about x-axis = $\sum ay^2$

Now M.I. of elementary area about Z-Z axis = az^2

$\therefore\ I_{zz} = \sum az^2 = \sum a(x^2 + y^2) = \sum ax^2 + \sum ay^2 = I_{yy} + I_{xx}$

$I_{zz} = I_{xx} + I_{yy}$

- ***MOMENT OF INERTIA OF RECTANGULAR LAMINA :***

Consider a rectangular section ABCD as in fig.

Let b = width of section, &

d = depth of section.

Now consider a strip 'PQ' of thickness 'dy' parallel to X-X axis & at a distance 'y' from it.

$\therefore$ Area of strip = b.dy

$\therefore$ Area of strip about X-X axis = area of strip . y^2 = b.dy.y^2 = b.y^2.dy

The M.I. of whole section can be found out by integrating for the whole length of lamina i.e. from –d/2 to d/2.

$$\therefore\ I_{xx} = \int_{-d/2}^{d/2} \text{b.y2.dy} = b\int_{-d/2}^{d/2} \text{y2.dy} = \left[by^3/3\right]_{-d/2}^{d/2}$$

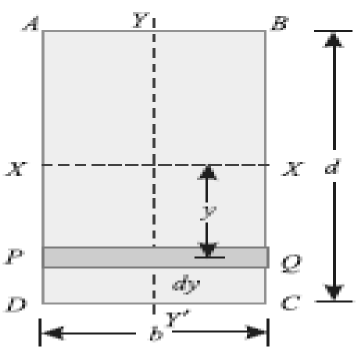
$$= b/3\left[(d/2)^3 - (-d/2)^3\right] = b/3\,(d^3/8 + d^3/8)$$

$$= 2bd^3/24 = \boldsymbol{bd^3/12}$$

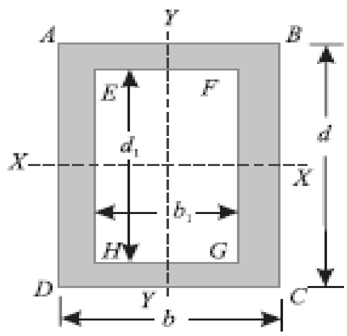
Similarly, $I_{yy} = db^3/12$

M.I. about the base, $I_{AB} = I_{XX} + Ay_c^2 = bd^3/12 + bd(d/2)^2$ since $y_c = d/2$

$= bd^3/12 + bd^3/4 = \boldsymbol{bd^3/3}$



Rectangular section.



Hollow rectangular section.

For hollow rectangle, $I_{XX} = (BD^3 - bd^3)/12$ & $I_{YY} = (DB^3 - db^3)/12$

- ***M.I. OF TRIANGULAR SECTION :***

Consider a triangular section ABC whose M.I. is to be required to be found out.

Let, b = Base width, & h = height.

Now consider a small strip, PQ, of thickness 'dx' at a distance of 'x' from A.

From the geometry of fig. Δ APQ & Δ ABC are similar.

$\therefore \frac{PQ}{BC} = \frac{x}{h}$ => $PQ = \frac{BC.x}{h} = \frac{b.x}{h}$

Area of strip, PQ = $\frac{b.x}{h}.dx$

M.I. of the base about the base, BC = Area x (distance)^2 = $\frac{b.x}{h}.dx \, . \, (h-x)^2 = \frac{b.x}{h}.(h\text{-}x)^2.dx$

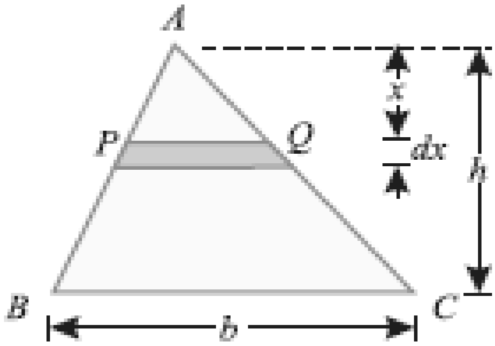
Now M.I. of whole triangular section may be found out by integrating for the whole height of the triangle (i.e. 0 to h)

$$I_{BC} = \int_0^h \frac{b\,x}{h}(h-x)^2\,dx$$

$$= \frac{b}{h}\int_0^h x\,(h^2 + x^2 - 2hx)\,dx$$

$$= \frac{b}{h}\int_0^h (xh^2 + x^3 - 2hx^2)\,dx$$

$$= \frac{b}{h}\left[\frac{x^2 h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3}\right]_0^h = \frac{bh^3}{12}$$



$I_{BC} = I_{GX} + Ay_c^2$ i.e. $I_{GX} = I_{BC} - Ay_c^2$ (from parallel axis theorem)

Here, y_c = distance between non-centoidal axis BC & axis X-X = h/3

$\therefore bh^3/12 = I_{GX} + ½\, bh\,(h/3)^2 = I_{GX} + bh^3/8$

$\therefore I_{GX} = bh^3/12 - bh^3/8 = bh^3/36.$

- ***M.I. OF A CIRCULAR SECTION :***

Consider a circle ABCD of radius (R) with centre 'O' and X-X' and Y-Y' be two axes of reference through 'O' as in fig.

Now consider an elementary ring of radius 'r' & thickness 'dr'.

Therefore area of the ring, da = $2\pi r.dr$

M.I. of the ring about the X-X axis & Y-Y axis = *Area x (distance)^2 = $2\pi r.dr \,.\, r^2 = 2\pi r^3.dr$*

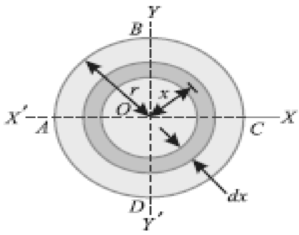
Now M.I. of the whole section about the central axis, can be found out by integrating the above equation for whole radius of circle i.e. from 0 to R.

$\therefore I_{ZZ} = \int_0^R 2\pi r^3.dr = 2\pi \int_0^R r^3.dr = \pi R^4/2 = \pi D^4/32$

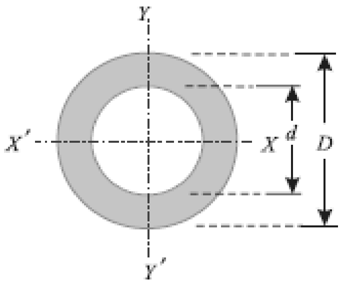
From theorem of perpendicular axis , we know that,

$I_{ZZ} = I_{XX} + I_{YY}$, but in case of circle, $I_{XX} = I_{YY}$

$\therefore$ ***$I_{XX} = I_{YY}$*** $= ½\, I_{ZZ} = ½ \,.\, \pi D^4/32 =$ **$\pi D^4/64$**



Circular section.



Hollow circular section.

- ***In case of hollow circle,***
- ***$I_{XX} = \pi D^4/64 - \pi d^4/64 = \pi/64\,(D^4 - d^4)$***
- ***M.I. OF A SEMICIRCLE :***

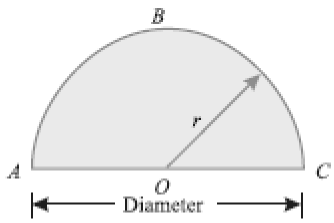
We know that, M.I. of circle about diametral axis AB, $I_{AB} = \pi D^4/64$

Therefore, M.I. of a semicircle about diametral axis,

I_{AB} = ½ $\pi D^4/64$ = ***$\pi D^4/128$ =***

Now the distance of centroidal axis from the diametral axis, $y_c = 4R/3\pi = 2D/3\pi$

Area of the semicircle, A = ½ . $\pi/4.D^2$ = 1/8 πD^2



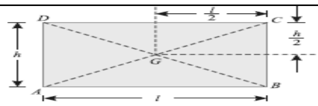
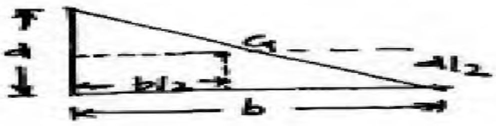
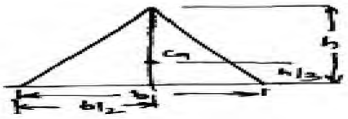
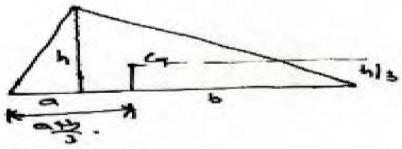
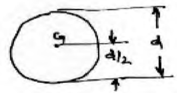
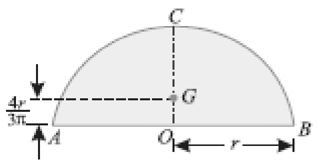
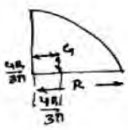
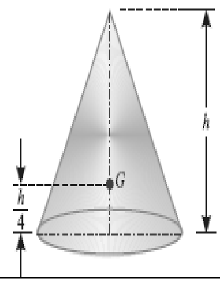
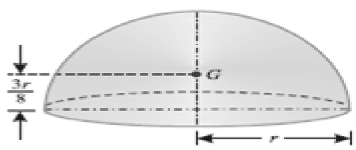
Semicircular section ABC.

From parallel axis theorem, $I_{AB} = I_{XX} + Ay_c^2$

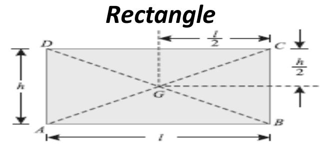
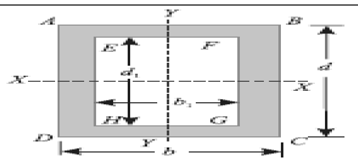
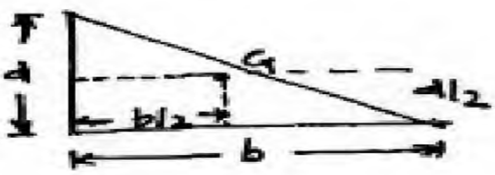
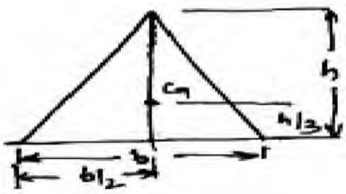
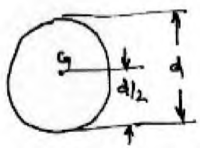
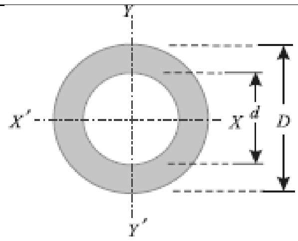
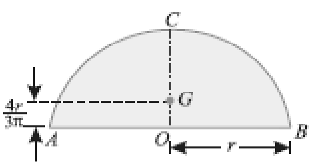
i.e. ***I_{XX}*** = $I_{AB} - Ay_c^2$

= $\pi D^4/128 - 1/8\,\pi D^2 \,.\, (2D/3\pi)^2$ = ***$0.0068598\, D^4 = 0.11\, R^4$***

- ***CENTROID OF AREAS & VOLUMES :***

Shape	***Figure***	***Area (A) / Volume (V)***	***Centroid 'G' (x_C, y_C)***
Plane Figure		$A = \int dA$	$x_C = \frac{\int dAx}{A}$ $y_C = \frac{\int dAy}{A}$
Rectangle	 Rectangle	$A = bd$	$x_C = b/2$ $y_C = d/2$
Triangle (right angled)		$A = ½ bh$	$x_C = b/3$ $y_C = h/3$
Triangle (isosceles)		$A = ½ bh$	$x_C = b/2$ $y_C = h/3$
Triangle (un-symmetric)		$A = ½ bh$	$x_C = (a + b)/3$ $y_C = h/3$
Circle		$A = \pi R^2$	$x_C = 0$ $y_C = 0$
Semicircle	 Semicircle	$A = ½ \pi R^2$	$x_C = 0$ $y_C = 4R / 3\pi$
Quadrant of a circle		$A = ¼ \pi R^2$	$x_C = 4R / 3\pi$ $y_C = 4R / 3\pi$
Right circular cone		$V = 1/3 \pi R^2 h$	$x_C = R$ $y_C = h/4$
Hemisphere		$V = 2/3 \pi R^3$	$x_C = R$ $y_C = 3R/8$

- ***MOMENT OF INERTIA ABOUT CENTROIDAL AXIS I_{XX}, I_{YY} :***

Figure	C (x_C & y_C) and area (A) x_C from left & y_C from base	I_{XX}, I_{YY} Centroidal axis
Rectangle  Rectangle	$x_C = b/2$ $y_C = d/2$ $A = bd$	$I_{XX} = bd^3 / 12$ $I_{YY} = db^3 / 12$ $I_{CD} = bd^3 / 3$ $I_{AD} = db^3 / 3$
 Hollow rectangular section.	$x_C = B/2$ $y_C = D/2$ $A = (BD – bd)$	$I_{XX} = 1/12 [BD^3 – bd^3]$ $I_{YY} = 1/12 [DB^3 – db^3]$
Right angled triangle 	$x_C = b/3$ $y_C = h/3$ $A = ½ bh$	$I_{XX} = bh^3 / 36$ $I_{YY} = hb^3 / 36$ $I_{BASE} = bh^3 / 12$
Equilateral & Isosceles triangle 	$x_C = b/2$ $y_C = h/3$ $A = ½ bh$	$I_{XX} = bh^3 / 36$ $I_{YY} = hb^3 / 48$ $I_{BASE} = bh^3 / 12$
Circle 	x_C = centre y_C = centre $A = \pi R^2$	$I_{XX} = I_{YY} = \pi D^4 / 64 = \pi R^4/4$ $I_{ZZ} = \pi D^4 / 32 = \pi R^4 / 2$
 Hollow circular section.	x_C = centre y_C = centre $A = \pi (R^2 – r^2)$	$I_{XX} = I_{YY} = \pi/4(R^4 – r^4)$ $I_{ZZ} = \pi/2 (R^4 – r^4)$
 Semicircle	$x_C = R$ $y_C = 4R / 3\pi$ $A = ½ \pi R^2$	$I_{XX} = 0.11 R^4$ $I_{YY} = I_{BASE} = \pi D^4/128 = \pi R^4/8 = 0.393 R^4$

SIMPLE MACHINES

- **MACHINE :** A *machine* is a device, which is capable of doing useful work. It transmits the energy supplied to it into useful amount of work.
- **SIMPLE MACHINE :** A *simple machine* may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.
- **COMPOUND MACHINE :** A *compound machine* may be defined as a device, consisting of a no. of simple machines, which enables us to do some useful work at faster speed or with a much less effort, as compared to a simple machine.
- **LIFTING MACHINE :** It is a device which enables us to lift a heavy load (W) by applying a comparatively smaller effort (P). **Ex**: Worm & worm wheel, Screw jack, Inclined plane, Levers.
- **WORKING PRINCIPLE OF LIFTING MACHINE :**

Let AB be a lever with its fulcrum at C.

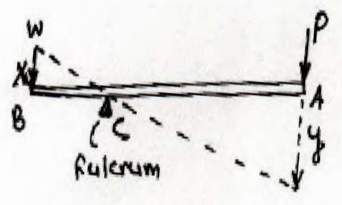
P = External force applied at end A

W = Load to be lifted by P

By taking moment about C of all the forces are in equilibrium,

P . AC = W . BC

But, AC > BC , Hence, W > P or P < W



So smaller amount of 'P' is applied at A to overcome a bigger amount of resistance 'W'.

- ***Important terms related to lifting machine :***

a. Mechanical advantages (M . A .) : The mechanical advantage (M.A) is the **ratio of weight lifted (W) to the effort applied (P)**. It is always expressed in pure no.

Mathematically, $M.A. = \frac{Weight\ lifted}{Effort\ applied} = \frac{W}{P}$

b. Input of a machine :

The input of a machine is the ***work done on the machine***. But the work is done on the machine by the effort. In a lifting machine, it is measured by the product of effort (P) & the distance through which it has moved.

Input = Effort applied (P) x Distance moved by effort (y) *i.e.* ***Input = P .y***

c. Output of a machine :

The output of a machine is the ***actual work done by the machine***. In a lifting machine, it is measured by the product of weight lifted (W) & the distance through which it has been lifted.

Output = Load lifted (W) x Distance through which it is lifted (x) i.e. **Output = W . x**

d. Velocity ratio (V.R.) :

It is defined as the **ratio of the distance moved by effort(y) to the distance moved by load (x).** It is always expressed in pure no..

Mathematically, $V.R. = \frac{Distance\ moved\ by\ effort}{Distance\ moved\ by\ load} = \frac{y}{x}$

It depends upon the dimension & physical features of the machine. Hence, Velocity ratio is constant for a particular lifting machine.

e. Efficiency of a machine (η) :

Efficiency of a machine is defined as **the ratio of output to the input** of the machine. It is generally expressed in percentage (%). It is denoted by the symbol ' η'.

$$Efficiency,\ \eta = \frac{Output}{Input} \times 100$$

- **IDEAL MACHINE :** If the efficiency of a machine is 100% i.e. if the output is equal to the input , the machine is said to be a *perfect* or *ideal machine.* In this case, **M.A. = V.R.**
- **RELATION BETWEEN EFFICIENCY, M.A. and V.R. :**

Consider a lifting machine, where W = Load lifted by machine

P = Effort required to lift the load

y = Distance moved by the effort, in lifting the load, and

x = Distance moved by load∴

$$\textbf{Efficiency},\ \eta = \frac{OUTPUT}{INPUT} = \frac{W.x}{P.y} = \frac{W}{P} \times \frac{x}{y} = \frac{W/P}{y/x} = \frac{M.A}{V.R}$$

Thus, the efficiency of a lifting machine is equal to the **ratio of the M.A. & V.R.** The values of M.A. & V.R. are equal, in case of machines whose efficiency is 100%. But in actual practice it is not possible.

- **REVERSIBILITY OF A MACHINE :**

Sometimes, a machine is also capable of doing some work, in the reverse direction, after the effort is removed. i.e. If the load falls down, by removing effort, the machine is said to be reversible machine. The action of such a machine is known as reversibility of the machine. In this case, work is done by the machine in reverse direction.

- **CONDITION FOR REVERSIBILITY OF A MACHINE :**

Consider a reversible machine, in which

Let, W = Load lifted by the machine

P = Effort required to loft the load

y = Distance moved by the effort, and

x = Distance moved by the load.

Then input of the machine = P .y

& output of the machine = W .x

We know that machine friction = input - output = P .y – W .x

A little consideration will show that, in a reversible machine, the output of the machine should be more than the machine friction when effort (P) is zero.

In other words, work done by load must be greater than the work lost in friction.

$$W.x > (P.y - W.x)$$

Or $\quad 2W.x > P.y$

Or $\quad \frac{W.x}{P.y} > \frac{1}{2}$ $\quad$ Or $\quad \frac{W}{P}.\frac{x}{y} > \frac{1}{2}$

Or $\quad \frac{W/P}{y/x} > \frac{1}{2}$ $\quad$ Or $\quad \frac{M.A.}{V.R.} > \frac{1}{2}$

Efficiency, $\eta > 0.5$ i.e. 50%

Hence, the condition for a machine, to be reversible, is that its ***efficiency should be more than 50%.***

- **SELF-LOCKING MACHINE :**

Sometimes, a machine is not capable of doing any work, in the reversed direction, after the effort is removed. Such a machine is called a non-reversible or self-locking machine.

In other words, if load doesn't fall down, by removing effort, the machine is said to be irreversible or self-locking machine.

- **Conditions for irreversibility :**

At irreversible stage, the load W will not fall down if the work done by the force friction is more than the output of the machine.

Work done by friction = P . y - W . x

The condition of irreversibility is

Work done by friction > output of the machine

So, $P.y - W.x > W.x$ $\quad$ *Or* $\quad P.y > 2W.x$

Or $2W.x < P.y$ $\quad$ *Or* $\quad \frac{W.x}{P.y} < ½$

$\eta < ½$ or 0.5 or 50% $\quad$ i.e. **Efficiency, $\eta < 50\%$**

Hence, the condition for a machine, to be irreversible, is that its ***efficiency should be less than 50%***

- **EFFORT LOST IN FRICTION : (Friction of a m/c in terms of effort)**

Let W = Load lifted and

P = Effort applied to lift the load (W), considering presence of friction in the machine.

P_1 = Effort required to lift the same load under ideal condition i.e. when there is no friction in the machine.

Then, Frictional force of the machine in terms of effort = Actual effort – Ideal effort = $P - P_1$ . . . . (1)

The velocity ratio of the machine is given by,

$$\text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{y}{x}$$

In frictionless condition, Input work = Output work i.e. $P_1 . y = P . x$

i.e. $P_1 = \frac{W.x}{y} = \frac{W}{\frac{y}{x}} = \frac{W}{V.R.}$

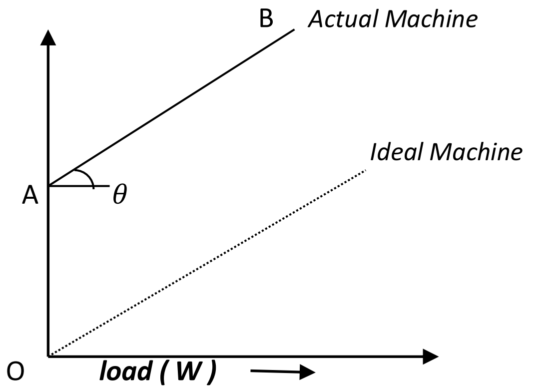
From equation (1), we get, **Effort lost in friction , $F_p = P - P_1 = P - \frac{W}{V.R.}$**

It means if the machine would be ideal (without friction) it would have required (**P** - $\frac{W}{V.R.}$) **N** less force to lift the same load (W).

- **FRICTION OF A MACHINE IN TERMS OF LOAD :**

Let P = Effort required to lift the load.
W = Actual load lifted by P
W_1 = Ideal load lifted by P &
F_w = Frictional force of machine in terms of load.

Frictional force of the machine in terms of load = Ideal load lifted – Actual load lifted
= W_1 - W



But, Efficiency = $\frac{Load}{Effort \ X \ V.R}$

For ideal machine, η = 100% = 1

i.e. 1 = $\frac{W1}{P.V.R}$

W_1 = P x V.R

F_w = W_1 - W = (P x V.R) - W

- **LAW OF THE MACHINE :**

The law of machine is defined by an equation, which gives the relationship between the effort required to raise the corresponding load.

OA = Amount of friction offered by machine.
= This is the effort required by machine to overcome the friction, before it can lift any load (i.e. under no load).

This graph we can get by experimenting on any lifting machine. By taking different load W_1, W_2, W_3, …….., W_n and knowing the corresponding efforts P_1, P_2, P_3, ……. , P_n a graph can be plotted by joining collinear points.

This graph is known as the law of machines.

Law of machine is **P = mW + C**

Where, P = Effort applied to lift the load,
W = Load lifted,
M = A constant (called co-efficient of friction), which equals to the slope of line AB.
C = Another constant, which represents the machine friction (i.e. OA in fig.)

- ***MAXIMUM EFFICIENCY OF A LIFTING MACHINE :***

Efficiency, $\eta = \frac{M.A.}{V.R.} = \frac{W/P}{V.R.} = \frac{W}{P.(V.R.)}$

For maximum efficiency, substituting the value of P = mW + C

$$\eta_{max} = \frac{W}{(mW + C).V.R} = \frac{1}{m.(V.R)} \quad \text{(neglecting } \frac{C}{W} \text{)}$$

- ***MAXIMUM M.A. OF A LIFTING MACHINE :***

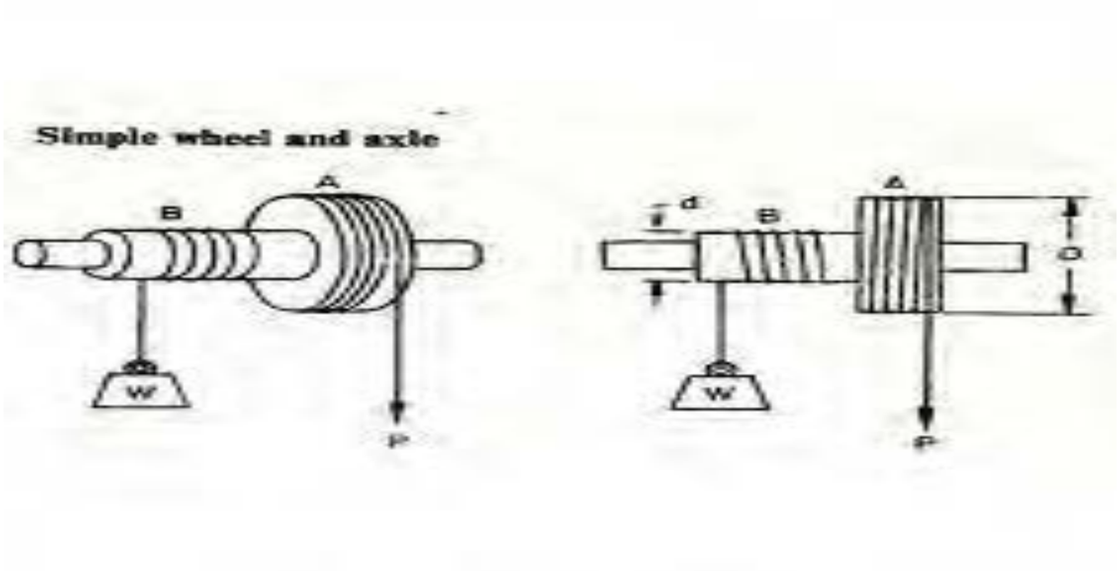
M.A. = W / P

For Maximum M.A. , substituting the value of P = mW + C

$$\textit{Maximum M.A.} = \frac{W}{mW + C} = \frac{1}{\frac{mW+C}{W}} = \frac{1}{m+\frac{C}{W}} = \frac{1}{m} \quad \text{(neglecting } \frac{C}{W} \text{)}$$

SIMPLE MACHINES

1. SIMPLE WHEEL & AXLE :



In a simple wheel & axle, the wheel 'A' & axle 'B' are keyed to the same shaft. The shaft is mounted on ball bearings, in order to reduce the frictional resistance to a minimum. A string is wound round the axle 'B', which carries the load W to be lifted. A second string is wound round the wheel 'A' in the opposite direction to that of the string on 'B'.

One end of the string is fixed to the wheel, while the other is free & the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of P will raise the load 'W'.

Let, D = Diameter of effort wheel,
d = Diameter of the load axle,
W = Load lifted &
P = Effort applied to lift the load.

Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution. We know that sdisplacement of the effort in one revolution of effort wheel 'A' = πD

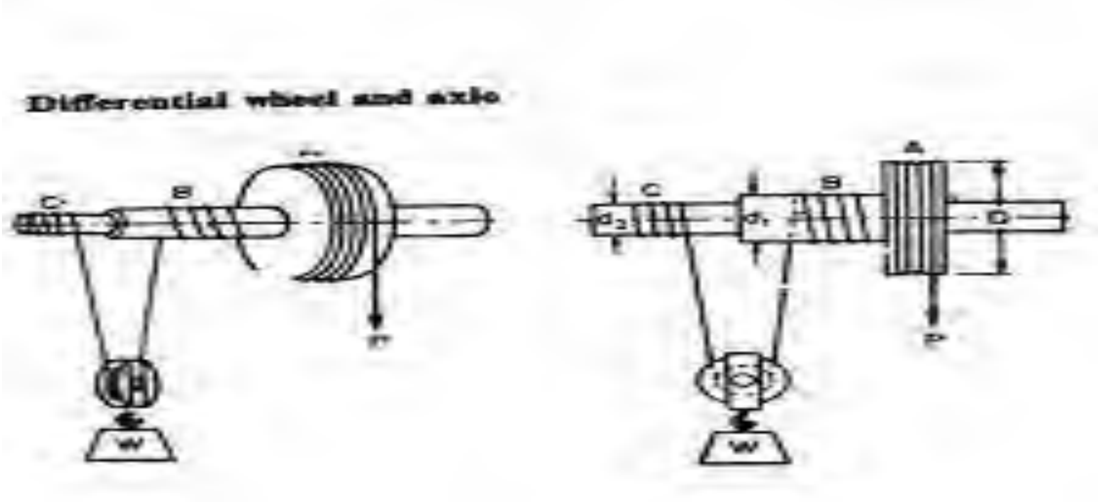
And displacement of the load in one revolution = πd

$\therefore$ V.R. $= \frac{\textit{Distance moved by the effort}}{\textit{Distance moved by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$

Now, M.A. = load lifted / Effort applied $= \frac{W}{P}$

And efficiency, $\eta = \frac{M.A.}{V.R.} = \frac{W/P}{D/d} = \frac{W.d}{P.D}$

2. DIFFERENTIAL WHEEL & AXLE :



It is an improved form of simple wheel & axle, in which the velocity ratio is intensified with the help of load axle. In this case, the load axle 'BC' is made up of two parts of different diameters. Like simple wheel and axle, the wheel 'A', and the axles 'B' & 'C' are keyed to the same shaft, which is mounted on ball bearings in order to reduce the frictional resistance to a minimum.

The effort string is wound round the wheel 'A'. Another string is wound round the axle 'B', which after passing round the pulley (to which the weight W is attached) is wound round the axle 'C' in opposite direction to that of the axle 'B'. *(care being taken to wind the string on the wheel 'A' and axle 'C' in the same direction)*

As a result of this, when the string unwinds from the wheel 'A', the other string also unwinds from the axle 'C'. But it winds on the axle 'B' as in fig.

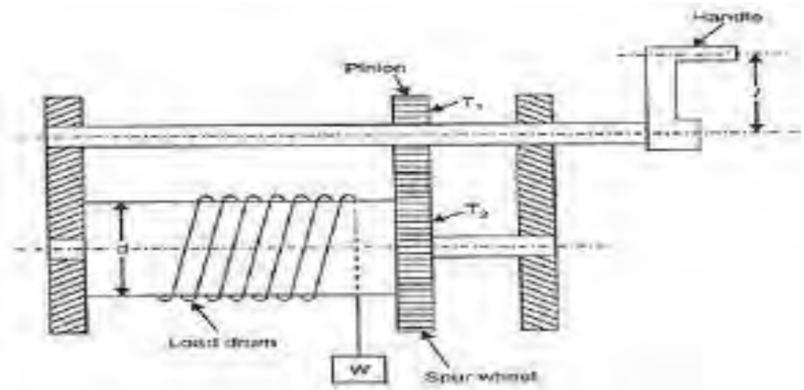
Let, D = Diameter of the effort wheel 'A',
d_1 = Diameter of the axle 'B',
d_2 = Diameter of the axle 'C',
W = weight lifted by the machine, and
P = Effort applied to lift the weight.

We know that displacement of the effort in one revolution of effort wheel 'A' = πD
Length of string, which will wound on axle 'B' in one revolution = πd_1
And length of string which will unwound from axle 'C' in one revolution = πd_2
$\therefore$ Length of string which will wound in one revolution = $\pi d_1 - \pi d_2 = \pi (d_1 - d_2)$
And displacement of weight = $\frac{1}{2}\pi(d_1 - d_2) = \frac{\pi}{2}(d_1 - d_2)$

$$\therefore \ \textit{V.R.} = \frac{\textit{Distance moved by the effort}}{\textit{Distance moved by the load}} = \frac{\pi D}{\frac{\pi}{2}(d1 - d2)} = \frac{2D}{(d1 - d2)}$$

Now M.A. = $\frac{W}{P}$ and Efficiency, $\eta = \frac{M.A.}{V.R.} = \frac{\frac{W}{P}}{\frac{2D}{(d1 - d2)}} = \frac{W(d1 - d2)}{2PD}$

3. <u>SINGLE PURCHASE CRAB WINCH</u> :



In single purchase crab winch, a rope is fixed to the drum & is wound a few turns round it. The free end of the rope carries the load W. a toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A as shown in fig.

The effort is applied at the end of the handle to rotate it.

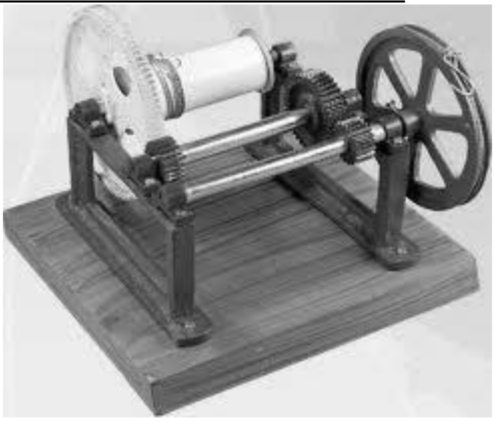
Let, T_1 = No. of teeth on the main gear (or spur wheel) A,
T_2 = No. of teeth on the pinion B,
l = Length of the handle,
r = Radius of the load drum
W = Load lifted, and
P = Effort applied to lit the load.

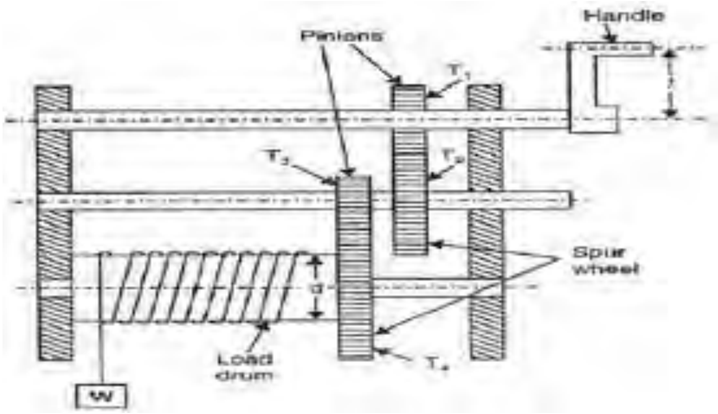
We know that, distance moved by the effort in one revolution of the handle = $2\pi l$
$\therefore$ No. of revolutions made by the pinion B = 1
And no. of revolutions made by the wheel A = T_2/T_1
$\therefore$ No. of revolutions made by the load drum = T_2/T_1
Distance moved by the load = $2\pi r \cdot T_2/T_1$

$$\therefore \text{V.R.} = \frac{\textit{Distance moved by the effort}}{\textit{Distance moved by the load}} = \frac{2\pi l}{2\pi r \cdot T2/T1}$$

Now M.A = $\frac{W}{P}$ and efficiency, $\eta = \frac{M.A}{V.R}$

4. **DOUBLE PURCHASE CRAB WINCH** :





A double purchase crab winch is an improved form of a single purchase crab winch, in which the velocity ratio I intensified with the help of one more spur wheel and a pinion. In a double purchase crab winch, there are two spur wheels of T_1 & T_3 as well as two pinions of teeth T_2 & T_4.

The arrangement of spur wheels & pinions are such that the spur wheel with T_1 gears with the pinion of teeth T_2. Similarly, the spur wheel with teeth T_3 gears with the pinion of the teeth T_4. The effort is applied to a handle as shown in fig.

Let, T_1 & T_3 = No. of teeth of spur wheels,
T_2 & T_4 = No. of teeth in the pinions.
l = Length of the handle,
r = Radius of the load drum,
W = Load lifted, and
P = effort applied to lift the load, at the end of the handle.

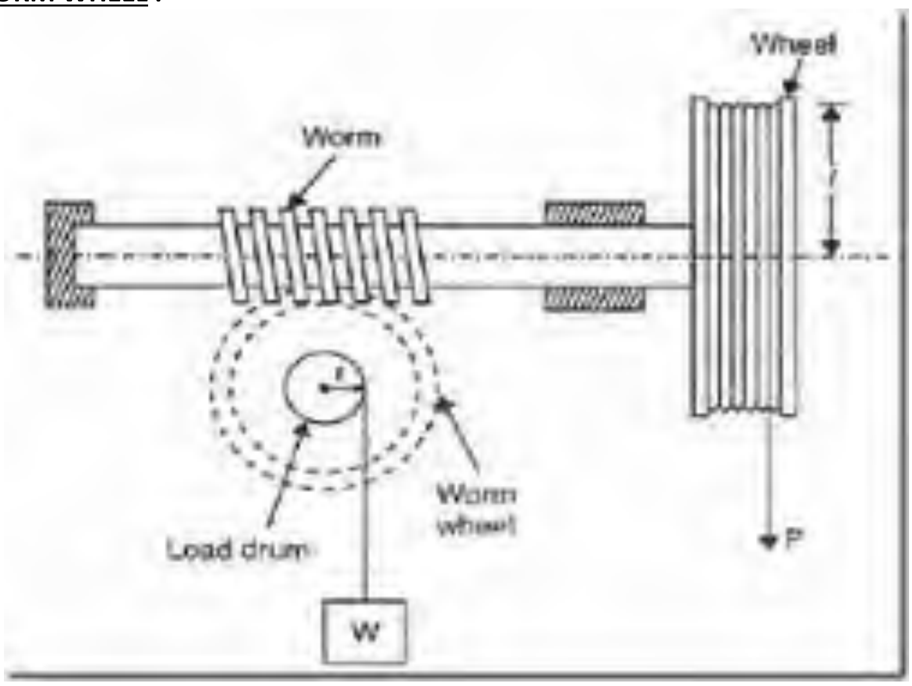
We know that, the distance moved by the effort in one revolution of the handle = $2\pi l$

∴ No. of revolutions made by the pinion 4 = 1
And no. of revolutions made by wheel 3 = T_4/T_3
∴ No. of revolutions made by the pinion 2 = T_4/T_3
And no. of revolutions made by wheel 1 = $T_2/T_1 \times T_4/T_3$
∴ Distance moved by the load = $2\pi r \times T_2/T_1 \times T_4/T_3$

$$\therefore \quad V.R = \frac{\textit{Distance moved by the effort}}{\textit{Distance moved by the load}} = \frac{2\pi l}{2\pi r \times T2/T1 \times T4/T3} = \frac{l}{r}(T_2/T_1 \times T_4/T_3)$$

Now **M.A = W/P** & **Efficiency, η = M.A./V.R.**

5. **WORM & WORM WHEEL** :



It consists of a square threaded screw, S (known as worm) and a toothed wheel (known as worm wheel) geared with each other, as shown in fig. A wheel A is attached to the worm, over which passes a rope. Sometimes, a handle is also fixed to the worm (instead of the wheel). A load drum is securely mounted on the worm wheel.

Let, D = Diameter of the effort wheel,
r = Radius of the load drum,
W = Load lifted,
P = Effort applied to lift the load,
T = No. of teeth on the worm wheel.

We know that distance moved by the effort in one revolution of the wheel (or handle) = πD

If the worm is single threaded (i.e. for one revolution of the wheel A, the screw S pushes the worm wheel through one teeth), then the load drum will move through = $\frac{1}{T}$ revolution

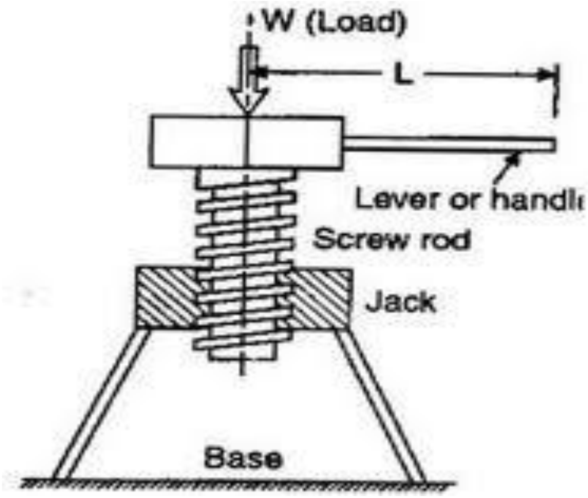
And distance, through which the load will move = $\frac{2\pi r}{T}$

$$\therefore \quad V.R. = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{\pi D}{\frac{2\pi r}{T}} = \frac{DT}{2r}$$

Now, **M.A.** = $\frac{W}{P}$ & **Efficiency,** $\eta = \frac{M.A.}{V.R.}$

1. *If the worm is double threaded, i.e. for one revolution of wheel A, the screw S pushes the worm wheel through two teeth, then V.R.* = $\frac{DT}{2 \times 2r} = \frac{DT}{4r}$
2. *In general, if the worm is 'n' threaded, then V.R.* = $\frac{DT}{n \times 2r}$

6. <u>SIMPLE SCREW JACK</u> :



It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane.

A simple screw jack, which is rotates by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.

Let, *l* = length of the effort arm,
p = pitch of the screw,
W = Load lifted, and
P = Effort applied to lift the load at the end of the lever.

We know that distance moved by the effort in one revolution of the screw = $2\pi l$

And distance moved by the load = p

$$\therefore \text{ Velocity ratio} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{p}$$

Now M.A. = $\frac{W}{P}$

$$\text{And Efficiency, } \eta = \frac{M.A.}{V.R.} = \frac{\frac{W}{P}}{\frac{2\pi l}{p}} = \frac{Wp}{P.2\pi l}$$

The value of P i.e. the effort may also be found out by the relation ***P* = W tan (** $\alpha + \varphi$ **)**

GEAR DRIVE

- **DEFINITION OF GEAR :**

Gear is a machine element which transmits power/torque between shafts, separated by small distance. Each gear is provided with projections called teeth & intermediate depression called tooth space. When two gears mesh, the teeth of one gear enter the spaces of the other. Thus the drive is positive & when one gear rotates, other also rotates.

- Gears are generally used for following reasons –
  - To reverse the direction of the rotation.
  - To increase or decrease the speed of rotation.
  - To move rotational motion to a different axis.
- ***ADVANTAGES & DISADVANTAGES OF GEAR DRIVE*** ***(compared to belt, rope & chain drive)***

ADVANTAGES :

- It transmits exact velocity ratio.
- It may be used to transmit large power.
- It has high efficiency.
- It has reliable service.
- It has compact layout.

DISADVANTAGES :

- The manufacture of gears require special tools & equipment.
- The error in cutting teeth may cause vibrations & noise during operation.
- It requires suitable lubricant & reliable method of applying it, for proper operation of gear drives.

- ***CLASSIFICATION OF GEARS :*** Gears are classified according to the following –

1. ***According to the position of axes of the shafts*** –

   The axes of two shafts between which the motion is to be transmitted, may be –

   a. Parallel - (spur gear, spur helical gear)
   b. Intersecting (non-parallel) - (bevel gear, helical bevel gear)
   c. Non-intersecting & non-parallel - (skew bevel gear or spiral gear)

a. Two parallel & co-planer shafts connected by the gears, are called ***spur gears*** & the arrangement is known as ***spur gearing***. These gears have teeth parallel to the axis of wheel.

In spur helical gearing, the teeth are inclined to axis. There are two types of helical gears such as – *single helical & double helical gears*. The double helical gears are known as ***herringbone gears***.

b. Two non-parallel or intersecting, but coplanar shafts connected by gears, are called ***bevel gears*** & the arrangement is known as bevel gearing. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, are known as ***helical bevel gears***.

- When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then bevel gears are known as ***mitres***.

c. Two non-intersecting & non-parallel i.e. non-coplanar shaft connected by gears, are called ***skew bevel gears*** or ***spiral gears*** & the arrangement is known as skew bevel gearing or spiral gearing.

2. According to the peripheral velocity of the gears –

The gears, according to the peripheral velocity of the gears may be classified as follows –

a. ***Low velocity*** - Gears having velocity less than 3 m/s
b. ***Medium velocity*** - Gears having velocity between 3 m/s & 15 m/s.
c. ***High velocity*** - Gears having velocity more than 15 m/s.

3. According to the type of gearing –

The gears, according to the type of gearing may be classified as follows –

a. External gearing,
b. Internal gearing &
c. Rack & pinion.

a. In external gearing, the gears of the two shafts mesh externally with each other. The larger wheel is called spur wheel & smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always ***unlike*** (opposite to each other).

b. In internal gearing, the gears of two shafts mesh internally with each other. The larger wheel is called annular wheel & smaller wheel is called pinion. In an internal gearing, the motion of two wheels is always ***like*** (same).

c. When a gear of a shaft meshes externally & internally with the gears in a straight line, such type of gear is called ***rack & pinion***. The straight line gear is called rack & the circular wheel is called pinion. With the help of rack & pinion, we can convert linear motion into rotary motion & vice-versa.

4. ***According to position of teeth on the gear surface –***

The teeth on the gear surface may be –

a. Straight teeth - spur gear have straight teeth.
b. Inclined teeth - helical gear have their teeth inclined to wheel rim.
c. Curved teeth - In spiral gears, the teeth are curved over rim surface.

- ***TERMS USED IN GEARS :***

- **Pitch circle –** It is an imaginary circle which by pure rolling action, would give the same motion actual gear.
- **Pitch circle diameter -** It is the diameter of the pitch circle. It is also known as pitch diameter.
- **Addendum –** It is the radial distance of a tooth from the pitch circle to the top of the tooth.
- **Dedendum -** It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
- **Addendum circle –** It is the circle drawn through the top of the teeth & is concentric with pitch circle.
- **Dedendum circle –** It is the circle drawn through the bottom of the teeth, it is also called as root circle.
- **Circular pitch –** It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding phone on the next tooth. It is usually denoted by p_C.

  Mathematically, $p_C = \pi D / T$

  Where, D = diameter of pitch circle.

  T = number of teeth on the wheel.
- ***Diametral pitch*** – It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by p_d. Mathematically, $p_d = T / D = \pi / p_C$
- ***Module –*** It is the ratio of pitch circle diameter in millimeters to the number of teeth. It is denoted by ***m***. Mathematically, $m = D / T$.
- ***Tooth space –*** It is the width of space between the two adjacent teeth measured along the pitch circle.
- ***Clearance –*** It is the radial distance from the top of the tooth to the bottom of the tooth. A circle passing through the top of the meshing gear is known as *clearance circle*.

- ***GEAR MATERIAL :***

The material used for the manufacture of gears depends upon the strength & service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel & bronze. The non-metallic materials like wood, compressed paper & synthetic resins like nylon are used for reducing noise.

- Cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability & ease of producing complicated shapes by casting method.
- The steel is used for high strength gears . The steel gears are usually heat treated in order to combine properly the toughness & tooth hardness.
- The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

- ***VELOCITY RATIO :***

Consider A & B are two pitch circles of driver gear & driven gear respectively.

Let N_A, N_B be the speed in r.p.m. of the driver & driven gears.

V_A, V_B be the linear velocity of driver & driven gears respectively.

D_A, D_B be the pitch circle diameters of driver & driven gears respectively.

$V_A = \omega_A D_A/2$ where, ω_A is the angular velocity of A.

$V_B = \omega_B D_B/2$ where, ω_B is the angular velocity of B.

At the point of contact 'C' of two gears the tangential velocity of the gears are same.

Hence, $V_A = V_B$ or $\omega_A D_A = \omega_B D_B$ or $\pi N_A D_A/60 = \pi N_B D_B/60$ or $N_A D_A = N_B D_B$ or $N_A/N_B = D_B/D_A$

Since module of both the mating gears are same, $m_A = m_B$

or $D_A/T_A = D_B/T_B$ where, T_A & T_B are the number of teeth of the gears A & B.

or $D_B/D_A = T_B/T_A$

∴ Velocity ratio (speed ratio) in terms of teeth, pitch circle diameters is given by

$$N_A/N_B = D_B/D_A = T_B/T_A$$

- ***GEAR TRAIN :***

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train*** or ***train of toothed wheels***.

The nature of train used depends upon the velocity ratio required & the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

- ***TYPES OF GEAR TRAINS :***

Following are the different types of gear trains, depending upon the arrangement of wheels

a. Simple gear train
b. Compound gear train
c. Reverted gear train
d. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

a. SIMPLE GEAR TRAIN :

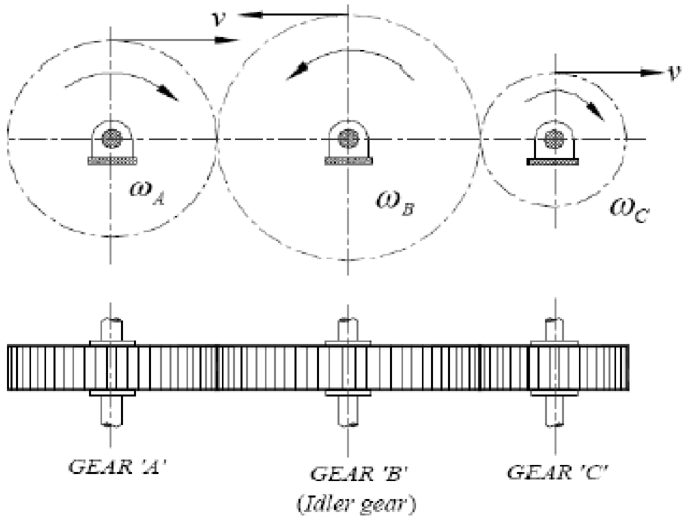
When there is only one gear on each shaft, it is known as ***simple gear train***. In a simple gear train all the gears lie on the same plane. In the gear train, gear A is ***driver*** & gear B is ***driven*** or ***follower***. The motion of the driven gear is opposite to the motion of driving gear.

Let N_1 = Speed of gear 1 (or driver) in rpm.
N_2 = Speed of gear 2 (or follower) in rpm.
T_1 = Number of teeth on gear 1,
T_2 = Number of teeth on gear 2.

∴ speed ratio of the gear train = $N_1 / N_2 = T_2 / T_1$

But***, train value*** is defined as the ratio of the speed of the follower to the ratio of the driver.

i.e. Train value = $N_2 / N_1 = T_1 / T_2$



Sometimes, the distance between two gears is large. The motion from one gear to another gear, in such a case, may be transmitted by either of following methods –

1. By providing the large sized gear, or
2. By providing one or more intermediate gears.

The former method (i.e. providing large sized gears) is very inconvenient & uneconomical method, where as the latter method (i.e. providing one or more intermediate gear) is very convenient & economical.

When the no. of intermediate gears are ***odd***, the motion of both the gears (i.e. driver & driven) is ***like***.

But if the no. of intermediate gears are ***even***, the motion of the driven will be in the ***opposite*** direction of the driver.

Now consider a simple train of gears with one intermediate gear,

Let, N_1 = Speed of driver in r.p.m.
N_2 = Speed of intermediate gear in r.p.m.
N_3 = Speed of the driven or follower in r.p.m.
T_1 = Number of teeth on driver

T_2 = Number of teeth on intermediate gear
T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is $N_1 / N_2 = T_2 / T_1$

Similarly as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is $N_2 / N_3 = T_3 / T_2$

The speed ratio of the gear train is obtained by –

$(N_1 / N_2) \times (N_2 / N_3) = (T_2 / T_1) \times (T_3 / T_2)$ or $N_1 / N_3 = T_3 / T_1$

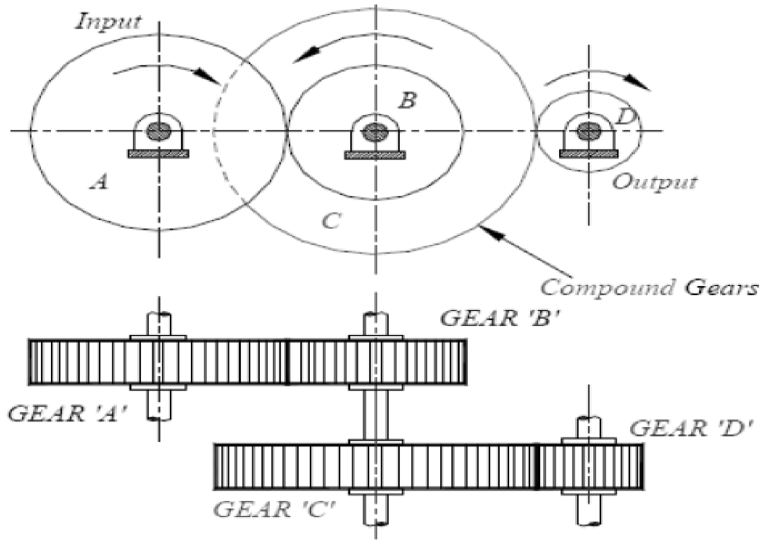
i.e. ***Speed ratio*** $= \dfrac{\text{speed of driver}}{\text{speed of driven}} = \dfrac{\text{No.of teeth on driven}}{\text{No.of teeth on driver}}$

Similarly, the above equation holds good even if there are any no. of intermediate gears. These intermediate gears are called ***idle gears***, as they do not effect the speed ratio of the gear system. The idle gears are used for the following two purposes –

1. To connect gears where a large centre distance is required, and
2. To obtain the desired direction of motion of the driven gear (i.e. clockwise or anti-clockwise).

b. COMPOUND GEAR TRAIN :

When there are more than one gear on a shaft, it is called a ***compound gear train***. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver & other with the driven or follower attached to the next shaft as in fig.



In a compound train of gears, the gear 1 is the driving gear mounted on shaft A, gear 2 & 3 are mounted on shaft B, gear 4 & 5 are mounted on shaft C & gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of the driving gear 1 in r.p.m.,
T_1 = Number of teeth on driving gear 1,
$N_2, N_3 \ldots N_6$ = Speed of respective gears in r.p.m.
$T_2, T_3 \ldots T_6$ = Number of teeth on respective gears .

Since gear 1 is in mesh with gear2, therefore the speed ratio is $N_1 / N_2 = T_2 / T_1$
Similarly, for gears 3 & 4, speed ratio is $N_3 / N_4 = T_4 / T_3$
And for gears 5 & 6, speed ratio is $N_5 / N_6 = T_6 / T_5$
The speed ratio of the compound gear train is obtained by
$(N_1 / N_2) \times (N_3 / N_4) \times (N_5 / N_6) = (T_2 / T_1) \times (T_4 / T_3) \times (T_6 / T_5)$
Or $N_1 / N_6 = (T_2 . T_4 . T_6) / (T_1 . T_3 . T_5)$

Since, gears 2 & 3 are mounted on the shaft B, therefore $N_2 = N_3$. Similarly gears 4 & 5 are mounted on shaft C, therefore $N_4 = N_5$.

i.e. ***speed ratio*** $= \dfrac{\text{speed of the first driver}}{\text{speed of the last driven or follower}} = \dfrac{\text{product of no.of teeth on he drivens}}{\text{product of no.of teeth on drivers}}$

The advantage of a compound gear train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.

The gears which mesh must have the same circular pitch or module. Thus gears 1 & 2, gears 3 & 4 and gears 5 & 6 have the same module as they mesh together.

DYNAMICS

- ***DYNAMICS :***
  It is that branch of engg. mechanics, which deals with the forces and their effects, while acting upon the body in motion.

Dynamics may be sub-divided into 2 branches-

a. ***Kinetics :*** It is the branch of dynamics, which deals with the bodies in motion due to the application of forces.
b. ***Kinematics :*** It is the branch of dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

- ***MOTION :*** When a rigid body changes its position with respect to the surroundings, it is said to be in motion.
- ***TYPES OF MOTION :***

When a body moves in space, it describes a curve, called path. This path may be straight, curved or circular.

a. ***Rectilinear motion –*** When a body moves along a straight line, its motion is called rectilinear motion. Motion of a freely falling body or a train running along a straight track etc. are examples of rectilinear motion.
b. ***Curvilinear motion –*** When a body moves along a curved line, its motion is called curvilinear motion. When an electric fan rotates, it executes rotational motion.
c. ***Circular motion –*** When a body moves along a circular path, its motion is called circular or rotary motion. Circular motion is a special case of linear motion.

- ***RECTILINEAR MOTION :***

1. **Distance –** It is the total route covered by a body during the motion. It is a scalar quantity. Its unit is cm, m, km.
2. ***Displacement –*** It is the shortest route covered by a body during the motion. It has both magnitude & direction. Hence, it is a vector quantity. It is denoted by (s). Its units are cm, m, km.
3. ***Speed –*** It is the rate of change of distance during motion.

   i.e. ***speed*** = $\frac{\textit{Distance}}{\textit{Time}}$

   It is a scalar quantity (as it is irrespective of its direction). It is expressed in cm/sec, m/sec & km/hr.

   (1 km/hr = $\frac{5}{18}$ m/s)

- If the moving particle passes over equal distances in equal interval of time, its speed is called ***uniform speed***.
- If a moving particle covers unequal distances in equal intervals of time, its speed is called ***variable speed***.
- If a particle moving in variable speed, then the average of all the variable speeds during the motion of the body, is termed as average speed.

  i.e. ***average speed*** = $\frac{\textit{Total distance moved}}{\textit{Total time taken}}$

4. ***Velocity –*** The rate of change of displacement during the motion of the body. As the velocity possesses both magnitude & direction, velocity is a vector quantity. It is denoted by 'v'. In M.K.S. & S.I. units it is usually expressed as m/s.

- If a moving particle passes through equal displacement in equal interval of time, its velocity is termed as ***uniform velocity***.
- If a moving particle passes through unequal displacement in equal interval of time, its velocity is termed as ***variable velocity***.
- If a particle is moving in variable velocity, then the average of all the variable velocities during the motion of the body, is termed as ***average velocity***.

  i.e. ***Average velocity*** = $\frac{\textit{Total displacement moved}}{\textit{Total time taken}}$

- The velocity of a particle at a particular instant of time is called ***instantaneous velocity***.

5. ***Acceleration –*** If the velocity of the body goes on changing at a uniform rate, then the rate of change of velocity is called acceleration. It is denoted by *'a'* & *'f'*.

   Mathematically, ***acceleration (a)*** = $\frac{\textit{Change in velocity}}{\textit{Time taken}}$

   $a = dv/dt = d/dt\,(ds/dt) = d^2s/dt^2$ (as $v = ds/dt$)

- **Units** – cm/s^2, m/s^2
- The uniform rate of increase in velocity is known as acceleration.
- The uniform rate of decrease in velocity is known as retardation.
- If the velocity of a moving body changes in equal magnitudes in equal intervals of time, it is said to be moving with a uniform acceleration. (i.e. acceleration remains constant)
- If the velocity of the moving body changes in unequal magnitudes in equal intervals of time, it is said to be moving with a variable acceleration (i.e. acceleration does not remains constant).
- ***MOTION UNDER UNIFORM ACCELERATION (Equations of linear motion) :***

Consider linear motion of a particle starting from O & moving along OX with a uniform acceleration. Let A be its position after 't' seconds.

O ——— ***A*** ——— ***X***

Let u = Initial velocity
v = Final velocity
t = Time (in seconds) taken by the particle to change its velocity from $\boldsymbol{u}$ to $\boldsymbol{v}$,
a = uniform positive acceleration, &
s = Distance travelled in $\boldsymbol{t}$ seconds.

Acceleration, a = Rate of increase in velocity = $\frac{v-u}{t}$ i.e. $v - u = a.t$

$$\therefore \boldsymbol{v = u + at} \quad . \ . \ . \ . \ . \ . \ . \ (1)$$

Average velocity = $\frac{u+v}{2}$

We know that, average velocity = Distance travelled by particle / time taken

i.e. Distance travelled by the particle = average velocity x time taken

$$s = \left(\frac{u+v}{2}\right).\, t$$

$\therefore s = \left(\frac{u+u+at}{2}\right).t$ i.e. $\boldsymbol{s = ut + \frac{1}{2} at^2}$ . . . . . (2)

From equation (1), we know that, $v = u + at$

i.e. $t = \frac{v-u}{a}$

Substituting the value of ' t ' in $s = \left(\frac{u+v}{2}\right).\, t$

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right) = (v^2 - u^2)/2a$

$or\ 2as = v^2 - u^2$ $\therefore \boldsymbol{v^2 = u^2 + 2as}$ . . . . . (3)

- ***DISTANCE TRAVELLED IN THE n_{th} SECOND :***

Consider the motion of a particle, starting from O and moving along OX.

Let u = Initial velocity of the particle,
v = Final velocity,
a = Constant acceleration
S_n = Distance (OB) travelled in ' n ' sec.
S_{n-1} = Distance (OA) travelled in (n – 1) sec.
S_{nth} = Distance (AB) travelled in n_{th} second
n = no. of second.

We know that distance travelled in ' t ' second ,$s = ut + \frac{1}{2} at^2$

Distance travelled in ' n ' seconds , $S_n = un + \frac{1}{2} an^2$

Distance travelled in (n – 1) seconds, $S_{n-1} = u(n-1) + \frac{1}{2} a(n-1)^2$

$\therefore$ distance travelled in n_{th} second, $S_{nth} = S_n - S_{n-1} = [un + \frac{1}{2} an^2] - [u(n-1) + \frac{1}{2} a(n-1)^2]$

$= un + \frac{1}{2} an^2 - un + u - \frac{1}{2} a(n^2 + 1 - 2n) = \frac{1}{2} an^2 + u - \frac{1}{2} an^2 - \frac{1}{2} a + an = u - \frac{1}{2} a + an$

$= u + a(n - \frac{1}{2}) = u + a/2(2n - 1)$

$$\therefore \boldsymbol{S_{nth} = u + \frac{a}{2}(2n - 1)}$$

- ***MOTION UNDER FORCE OF GRAVITY :***

When a body falls down, its velocity gradually increases. From this phenomenon it can be concluded that the falling bodies are subjected to acceleration. During falling, a body is under the action of gravitational force of the earth, it will move with some acceleration. This acceleration is called 'acceleration due to gravity ' & is always denoted by ' g '. The value of acceleration due to gravity is varies from 9.77 m/s^2 to 9.83 m/s^2 over the world. But the average value is 9.807 m/s^2 (say 9.81 m/s^2).

In case of vertical motion under gravity, 'a' will be replaced by 'g' & 's' will be replaced by 'h'.

When a body falls under gravity, the following formulae will be applicable :

$$v = u + gt$$
$$h = ut + ½\,gt^2$$
$$v^2 = u^2 + 2gh$$
$$h_{nth} = u + \frac{g}{2}\,(2n - 1)$$

When a body is thrown vertically upward, the following formulae are applicable :

$$v = u - gt$$
$$h = ut - ½\,gt^2$$
$$v^2 = u^2 - 2gh$$
$$h_{nth} = u - \frac{g}{2}\,(2n - 1)$$

- ***TIME OF RISE OF A BODY PROJECTED VERTICALLY UPWARD :***

Let a body thrown vertically upward. After reaching its maximum height its velocity becomes to zero & at that point the body begins to fall.

Let t = Required time of rise
u = Initial velocity with which the body is projected upward,
v = Final velocity of the body (= 0)
g = Acceleration due to gravity.

According to formulae, $v = u - gt$ *i.e* $0 = u - gt$ *or* $\mathbf{t = u/g}$

- ***GREATEST HEIGHT ATTAINED BY A BODY PROJECTED VERTICALLY UP :***

Let H = required greatest height attained by a body
u = Initial velocity with which the body is projected upward, and
g = acceleration due to gravity.

At the greatest height the vertical upward velocity (i.e. final velocity) of the projected body becomes nil.

Hence, according to formula, $v^2 = u^2 - 2gh$ *, we get*
$0 = u^2 - 2gH$ *or* $\mathbf{H = u^2/2g}$

- ***TIME OF RISE & TIME OF FALL FOR A BODY PROJECTED VERTICALLY UPWARD :***

In Upward Motion :

u = initial velocity
v = final velocity (i.e. v = 0 at the highest point)
t_1 = time taken by the body to reach the highest point.
$v = u - gt_1$ *i.e.* $0 = u - gt_1$ *i.e.* $\mathbf{t_1 = u/g}$
$v^2 = u^2 - 2gh$ *i.e.* $0 = u^2 - 2gh$ *i.e.* $h = u^2/2g$ . . . . (1)

In Downward Motion :

$u = 0$
t_2 = time taken by the body to reach point of projection from maximum height,
v = final velocity
$v = u + gt_2$ *i.e* $v = gt_2$ (*as* $u = 0$)
$v^2 = u^2 + 2gh$ *i.e.* $(gt_2)^2 = 0 + 2gh$ *i.e.* $h = ½\,gt_2^2$ . . . . (2)

From equation (1) & (2), we get
$u^2/2g = ½\,gt_2^2$ *i.e.* $t_2^2 = u^2/g^2$ *i.e.* $\mathbf{t_2 = u/g}$
$\therefore\ \mathbf{t_1 = t_2 = u/g}$

Therefore, time of rise is equal to the time of fall for a body projected vertically upwards.

Time of flight = Time of rise + Time of fall = u/g + u/g = 2u/g

- ***ROTATIONAL MOTION :*** If a body revolves about a fixed point or axis its motion is said to be rotational. As the body rotates, the line joining any two points in it continuously changes the angle with its initial direction.
- ***ANGULAR DISPLACEMENT (θ) :*** Angular displacement of a particle during any time period is the angle through which the particle turns. It is always expressed in radians & is denoted by ‘ ***θ*** ‘.
- ***ANGULAR VELOCITY (ω) :***

Angular velocity of a particle is defined as the rate at which a particle turns through an angle in radians per second. It is denoted by ‘ ***ω*** ’.

Let a particle move in a circular path of radius r about the centre O. If the particle moves from A to B in t seconds, the angular displacement of the particle in t seconds is ∠AOB i.e. ***θ*** radians.

So, the angular velocity of the particle is given by, $\omega = \frac{\text{Angular displacement}}{\text{Time}} = \frac{\theta}{t}$

- ***RELATION BETWEEN LINEAR VELOCITY & ANGULAR VELOCITY :***

Let v = linear speed of a particle in m/sec moving along a circular path. The direction of v is tangent to the circle.

r = radius of the circlar path in metre,

N = r.p.m. of thr particle,

ω = angular velocity of the particle in radians/sec,

In one rotation of the particle, it describes a distance of $2\pi r$ metres

In N rotations (i.e. in one minute), the particle describe a distance of $2\pi rN$ metre.

In one second, the particle describes a distance of $2\pi rN/60$ metre.

Hence, $v = 2\pi rN/60$ m/sec

But, $\omega = 2\pi N/60$ rad/sec

From the above two equations, we get, $\boldsymbol{v = \omega r}$

Uniform Angular Velocity : If a particle describes equal angular displacement in equal intervals of time, it is said to be moving with uniform angular velocity.

Variable Angular Velocity : If a particle doesn't describe equal angular displacements in equal inter intervals of time, it is said be moving with variable angular velocity (or non-uniform angular velocity).

Angular Acceleration : It is the rate of change of angular velocity & is expressed in rad/sec^2. It is denoted by ' α '.

Angular acceleration = change in angular velocity / time

i.e. $\alpha = (\omega - \omega_0)/t$

where, ω_0 = Initial angular velocity in rad/sec

ω = Final angular velocity in rad/sec

t = time taken.

- ***EQUATIONS OF CIRCULAR MOTION :***

1. $\omega = \omega_0 + \alpha t$
2. $\theta = \omega_0 + \frac{1}{2}\alpha t^2$
3. $\omega^2 = \omega_0 + 2\alpha\theta$

- ***RELATION BETWEEN LINEAR ACCELERATION (a) & ANGULAR ACCELERATION (α) :***

We know that, $\mathbf{v = u + at}$

$\omega = \omega_0 + \alpha t$

$v = \omega r$

$u = \omega_0 r$

$v = u + at$ i.e. $\omega r = \omega_0 r + at$ i.e. $(\omega_0 + \alpha t) r = \omega_0 r + at$ i.e. $\omega_0 r + \alpha t r = \omega_0 r + at$

i.e. $\alpha t r = at$ i.e. $a = \alpha r$

- ***LAWS OF MOTION :***

There are three laws of linear motion which governs the equations of kinetics, formulated by *Sir Issac Newton* in *1680*.

- ***Terms used -***

Momentum : It is the total motion possessed by a body. It can be expressed mathematically as

Momentum = mass x velocity

The unit of momentum in S.I. system is kg m/sec

Inertia : It is an inherent property of a body, which offers resistance to the change of its state of rest or uniform motion.

- ***NEWTON'S FIRST LAW OF MOTION :***

It states that, *" Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."* It is also called law of inertia & consists of the following two parts :

a. A body at rest continues in the same state, unless acted upon by some external force. This tendency is known as *inertia of rest*. A book lying on the table remains at rest, unless it is lifted or pushed.

b. A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compiled by some external force to change its state. This tendency is known as *inertia of motion*.

- ***NEWTON'S SECOND LAW OF MOTION :***

It states that *" The rate of change of momentum is directly proportional to the impressed force & takes place, in the same direction in which the force acts."*

This law enables us to measure a force & establishes the fundamental equation of dynamics.

Consider a body moving in a straight line.

Let its velocity be changed while moving.

Let m = mass of the body, a = constant acceleration,
u = initial velocity of the body, v = final velocity of the body,
t = time (in sec.) required to change the velocity from u to v, &
F = force required to change the velocity from u to v in t second.

Initial momentum = mu

& final momentum = mv

$\therefore$ Rate of change of momentum = $\frac{mv-mu}{t} = \frac{m\,(v-u)}{t} = ma$ . . . (as $\frac{v-u}{t} = a$)

According to Newton's second law of motion, the rate of change of momentum is directly proportional to the impressed force.

$\therefore F \propto ma = kma$

Where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted in such that it produces unit acceleration to a unit mass.

$\therefore$ F = ma = mass x acceleration.

- ***NEWTON'S THIRD LAW OF MOTION :***

It states *" To every action, there is always an equal & opposite reaction."*

Action means the force, which a body exerts on another, & the reaction means the equal & opposite force, which the second body exerts on first body. This law, therefore, states that a force always occurs in pair. Each pair consisting of two equal & opposite forces.

Example – When a swimmer tries to swim, he pushes the water backwards & the reaction of the water pushes the swimmer forward.

- ***RECOIL OF GUN :***

According to Newton's third law of motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as recoil of gun.

Let M = mass of the gun, m = mass of the bullet, &
V = velocity of the gun with which it recoils, v = velocity of the bullet after explosion.

$\therefore$ momentum of bullet after explosion = mv

And momentum of gun = MV

Equating the above two momentums, we get MV = mv i.e. ***V = mv / M***

This relation is known as Law of conservation of momentum.

- ***MOTION OF A BOAT :***

If the boat is at rest & the boat boy runs on it & dives off into the water, the boat will also move backward.

Now consider a boat on which a boat boy runs & then dives off into the water.

Let M = mass of the boat,
V = velocity of the boat,
m = mass of the boat boy, &
v = velocity of the boat boy.

Momentum of the boat, after the boy jumps = MV

Momentum of the boat boy = mv

Equating the above two momentums, we get MV = mv i.e. ***V = mv / M***

- ***MOTION OF A LIFT / ELEVATOR :***

Consider a lift / elevator, carrying some mass & moving with a uniform acceleration.

Let, m = Mass carried by the lift,
a = Uniform acceleration of the lift, and
R = Reaction of the lift or tension in the cable, supporting the lift.

a. When the lift is moving upwards :

We know that downward force due to mass of the lift = mg

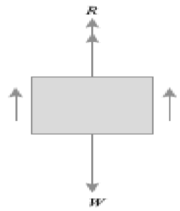
And net upward force on lift, F = R – mg . . . (1)

We also know that, this force

= mass x acceleration = m . a . . . (2)

Equating equations (1) & (2), we get -

R – mg = ma or R = ma + mg = m(a + g)



(a) Lift moving upwards

b. When the lift is moving downwards :

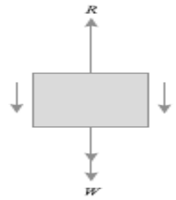
In this case, the net downward force, which is responsible for the motion of the lift,

F = mg – R . . . . . (3)

From equations (1) & (3),

ma = mg – R

or R = mg – ma = m(g – a)



(b) Lift moving downwards

- In the above cases, we have taken mass carried by the lift only. We have assumed that, it includes mass of lift also. But sometimes, for example contains mass of the lift and mass carried by the lift separately.

  In such a case, the mass carried by the lift (or mass of man/operator) will exert a pressure on the floor of the lift. Whereas tension in the cable will be given by the algebraic sum of the masses of the lift & mass carried by the lift.

- **When the lift is moving upward :**

  The pressure exerted by the mass carried by the lift (man) on the floor , F = $m_2(g + a)$

  And tension on the cable, R = $(m_1 + m_2)(g + a)$

- **When the lift is moving downward :**

  The pressure exerted by the mass carried by the lift (man) on the floor , F = $m_2(g - a)$

  And tension on the cable, R = $(m_1 + m_2)(g - a)$

  Where, m_1 = Mass of the lift, m_2 = mass of the man (mass carried by the lift)

  $m = m_1 + m_2$

- **PILE & PILE HAMMER :**

  Let, M = Mass of the pile hammer,

  m = Mass of the pile,

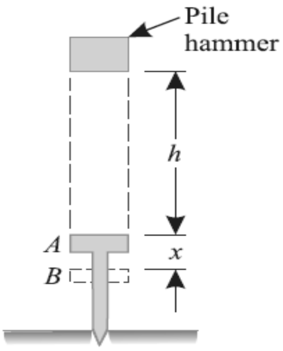
  h =Height through which the pile hammer falls before striking pile,

  x = Distance through which pile is driven into ground / blow,

  v = Velocity of the pile hammer just before impact = $\sqrt{2gh}$

  resistance offered by the ground,

  $R = M^2gh / [x(M + m)] + (M + m)g$



- ***D'ALEMBERT'S PRINCIPLE :***

It states that, *the sum of the resultant of a system of forces acting on a body having rectilinear motion, & the inertia force (or reversed effective force) is zero*. i.e. $\boldsymbol{P + P_i} = 0$

Let ***P*** be the resultant of a no. of forces acting on a body having a mass of '***m***' kg.

Then P = ma Newton. Where '***a***' is the acceleration in m/s^2 generated in the body.

Now if force of equal magnitude '***P***' be applied to the body in opposite direction, then ***P – ma = 0***. This is called equation of dynamic equilibrium for rectilinear motion.

In order to write down this equation, we are to consider, in addition to real forces acting on a body, an imaginary force (***- ma***). This force is called inertia force or reversed effective force.

If P_i denotes inertia force, we can write $\boldsymbol{P + P_i = 0}$

- While applying D'Alemberts Principle, following points should be carefully noted –

  1. The inertia force P_i doesn't actually act on a body. It is an imaginary force.

  2. Since $\boldsymbol{P_i = ma}$, the point of application of P_i is the centre of mass of the body or centre of gravity of the body.

- ***Conditions :***
  1. The algebraic sum of the resolved parts of the given forces, actually acting on the body & the inertia force P_i along any direction is zero.
  2. The algebraic sum of the resolved parts of the given forces actually acting on the body & the inertia force P_i along a direction perpendicular to the previous direction is zero.
  3. The algebraic sum of the moments of the forces actually acting on the body & the inertia force P_i about any point or axis is zero.

- **WORK, POWER & ENERGY :**
- **WORK :**

Whenever a force acts on a body, & the body undergoes some displacement, then work is said to be done. i.e. If a force F, acting on a body, causes it to move through a distance 's', then work done by the force, F = Force x Distance = F x s.

Sometimes, the force F doesn't act in the direction of motion of the body (i.e. the body doesn't move in the direction of force, the work done by the force F

= component of force in the direction of motion x distance = $F \cos\theta \cdot s$

- ***WORK DONE ON A INCLINED PLANE :***

A body having a weight W slides down a smooth inclined plane of inclination θ with the horizontal.

∴ work done by $W = W \sin\theta \cdot s$

Where, W sin θ = component of W in the direction in which motion takes place,

s = distance through which the body moves down in the inclined plane.

- ***WORK DONE BY A FORCE & WORK DONE AGAINST A FORCE :***

If the point of application of the force acting on a body moves in the direction in which the body moves, work is said to be *done by the force.*

If the point of application of the force acting on a body moves in the direction opposite to that in which the body moves, work is said to be *done against the force*.

Work done by the force is treated as ***+ ve*** work &

work done against the force is treated as ***– ve*** work.

Units :

The units of work done are ***N m*** (joule), ***kN m*** (kilo joule)

One N m is the work done by a force of 1 N, when it displaces the body through 1 m. It is called 1 joule. (1 joule = 1 N m)

- Sometimes, the force stretches or compresses a spring or penetrates into a body. In such a case, *the average force is taken as half of the force for the purpose of calculating work done*.

  i.e. ***work done in spring = average force x distance = (force required to stretch the spring / 2) x distance***

- ***POWER :*** The power may be defined as the rate of doing work. It is thus the measure of performance of engines e.g. an engine doing a certain amount of work, in one second, will be twice as powerful as an engine doing the same amount of work in two seconds.

Units : In S.I. units the unit of power is watt (briefly written as W)

1 Watt or 1 joule/sec = 1 N m/sec. The bigger units are ***kW*** (10^3 W), ***MW*** (10^6 W).

- ***TYPES OF ENGINE POWER :***

  In engines, following two terms are used for power –

  a. ***Indicated power :*** The actual power generated in the engine cylinder is called indicated power (I.P.).
  b. ***Brake power :*** The net output of the engine (i.e. I.P. – losses) is called brake power (B.P.)

- ***EFFICIENCY OF AN ENGINE :*** It is also called mechanical efficiency of an engine. It is the ratio of brake power to the indicated power.

  Mathematically, efficiency, $\eta = \frac{\text{B.P.}}{\text{I.P.}}$

- ***MEASUREMENT OF BRAKE POWER :***

  a. ***Brake power*** $= \frac{(w-s)\,\pi\,(D+d)\,N}{60}$ watt - (Rope brake dynamometer.)

  If diameter of the rope not considered, then B.P. $= \frac{(w-s)\,\pi\, D\; N}{60}$ **watt**

  Where, *w* = Dead load,

  s = spring balance reading,

  N = speed of the engine shaft in r.p.m.,

  D = Effective diameter of the flywheel.

b. ***B.P.*** = $\frac{2\,\pi\,N\,(\,W\,.L\,)}{60}$ watt

Where, W = weight hung from the lever,

L = horizontal distance between the centre of the pulley & the line of action of the weight (W)

N = speed of the engine shaft in r. p.m.

c. ***B.P.*** = $\frac{2\pi\,N\,T}{60}$ watt where, T = torque.

- ***ENERGY :*** The energy may be defined as the capacity to do the work. Its units are same as unit of work.

Mechanical Energy : Following two types are important from subject point of view –

1. Potential energy ***2.*** Kinetic energy.

1. Potential Energy : It is the energy possessed by a body, for doing work, by virtue of its position.

Consider a body of mass (m) raised through a height (h) above the ground level. We know that work done in raising the body = weight x distance = mg . h = mgh

This work (= mgh) is stored in the body as potential energy. The body while comong down to its original level, is capable of doing work equal to mgh.

a. A body, raised to some height above the ground level, possesses some potential energy, because it can do some work by falling on the earth's surface.

b. Compressed air also possesses potential energy, because it can do some work in expanding, to the volume it could occupy at atmospheric pressure.

c. A compressed spring also possesses potential energy, because it can do some work in recovering to its original shape.

2. Kinetic Energy : It is the energy possessed by a body, for doing work by virtu of its mass & velocity of motion.

Consider a body, which has been brought to rest by a uniform retardation due to the applied force.

Let m = mass of the body,

u = initial velocity of the body,

P = force applied on the body to bring it to rest,

a = constant retardation &

s = distance traveled by the body before coming to rest.

Since the body is brought to rest, therefore its final velocity, V = 0.

Work done, W = force x distance = P . s = m.a.s (as P = ma)

We know that, $V^2 = u^2 - 2as$ (- ve sign due to retardation)

i.e. $2as = u^2$ or $as = u^2 / 2$

Work done = Kinetic energy = mas = m . u^2 / 2 = ½ mu^2

If we take initial velocity as v, then ***K.E. = ½ mv²***

- ***SIMPLE HARMONIC MOTION :***

The to & fro motion of a body about a fixed point is called simple harmonic motion, where the acceleration is proportional to the displacement & is directed towards the centre.

Consider a particle starting from A & moving round the circumference of circle in anticlockwise direction, with a constant angular velocity (as in fig.).

Let P = position of the particle at any instant , &

N = projection of P on the diameter Y – Y' of the circle.

When the point ' P ' moves round the circumference of the circle from A to B, N moves from O to B. When P moves from 'B to C ', N moves from ' B to O '. Similarly 'P' moves from ' C to D ', N moves from ' O to D ' and finally P moves from ' D to A ' N moves from ' Y' to O '. Hence as P completes one revolution, the point N completes one vibration about the point O. This to & fro motion of N is known as " Simple Harmonic Motion ".

- ***CONDITIONS OF S.H.M. :***

A body is said to move or vibrate with S.H.M., if it satisfies the following condition –

a. Its acceleration is always directed towards the centre, known as point of reference or mean position.

b. Its acceleration is proportional to the distance from that point.

Oscillation : It is one complete vibration of a body. When the body moves from ***y to y'*** & back to ***y***, it is said to have complete one oscillation.

Beat : It is the half of the oscillation, when the body moves from ***y to y'*** or ***y' to y***, it is said to have complete one beat.

Amplitude : It is the maximum displacement of a body from its mean position. In the above fig. ***OY & OX*** is the amplitude of the particle N. It is equal to the radius of the circle.

Time Period : In general, the harmonic motion are periodic, i.e. they repeat themselves in equal intervals of time. The interval of time taken by the motion to repeat itself is called a period. It is the mean time taken by a particle for one complete oscillation.

Mathematically, time period can be written as $T = 2\pi / \omega$

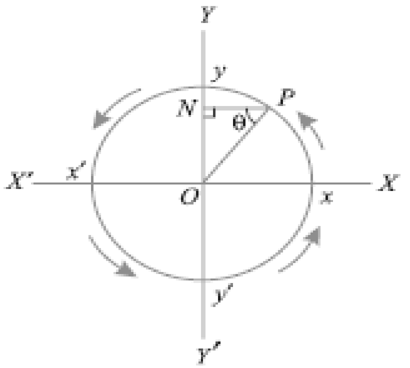
Where, ω = angular velocity of the particle in radians/sec.

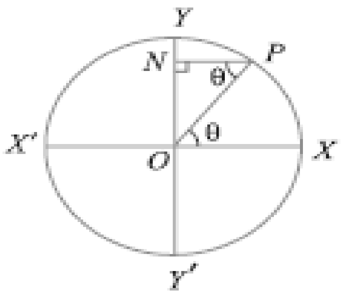
Time period of a simple harmonic motion (SHM) is independent of amplitude.

Frequency : The number of cycles of motion completed in a unit interval of time is known as frequency.

Frequency = 1 / T where, T = time period.

Units of frequency is ***Hertz*** *(Hz) or* ***cycle per second***.





Particle moving with S.H.M.

- ***VELOCITY & ACCELERATION OF A PARTICLE MOVING WITH S.H.M.***

Consider a particle moving along the circumference of a circle, of radius 'r' with a uniform angular velocity of ω rad / sec.

Let P be the position of the particle at some instant after t seconds from x, there angle turned by the particle, $\theta = \omega t$ rad.

Displacement of the point N, i.e. projection of P on the vertical diameter Y-Y of the circle,

$y = ON = r \sin\theta$

$y = r \sin \omega t$ . . . . (1)

Differentiating this equation with respect to time 't'

$dy / dt = v = r\omega \cos \omega t$ . . . . . . (2)

or velocity, $v = r \omega \sqrt{1 - \sin^2 \omega t}$

From equation (1) we get, $\sin \omega t = y / r$

Substituting this value of $\sin \omega t$ in the equation (2)

$dy / dt = v = r\omega \sqrt{1 - y^2/r^2}$ or $v = \omega \sqrt{r^2 - y^2}$ . . . . . . (3)

Now differentiating equation (2) with respect to time 't'

$D^2y/dt^2 = - r \omega^2 \sin \omega t$ . . . . (4)

Or $\mathbf{a = - \omega^2 y}$

In S.H.M., acceleration is directly proportional to displacement & is directed opposite to the displacement.

Negative sign indicates that the direction of acceleration is opposite to the direction of displacement i.e. it is always directed towards ' O '. But in actual practice, this relation is used as

$\mathbf{a = \omega^2 y}$.

- **MAXIMUM VELOCITY & ACCELERATION OF A PARTICLE MOVING WITH SHM :**

The velocity of a particle moving in simple harmonic motion is

$v = \omega \sqrt{r^2 - y^2}$ . . . . . . (1)

It shows that the velocity is maximum, when y = 0 or when N passes through 0, i.e., its mean position.

Therefore maximum velocity, $v_{max} = \omega r$ . . . . (2)

It may be noted from equation (1) that its velocity is zero when y = r when N passes through B or D as in fig. i.e., at extreme positions.

The acceleration of the particle moving with the S.H.M. is $a = -\omega^2 y$ . . . . (3)

It shows that the acceleration is maximum when the value of y is maximum i.e. y = r, & N passes through B or D. Therefore maximum acceleration, $a_{max} = -\omega^2 r$ . . . . . (4)

Equation (4) shows that the direction of acceleration is opposite to the direction in which y increases i.e. the acceleration is always directed towards 'O'.

It may also be noted from equation (3) that the acceleration is zero, when y = 0 or when N passes through 'O' i.e. its mean position. It is thus obvious that the acceleration is proportional to the distance from 'O', i.e., mean position.

- ***CONSERVATION OF ENERGY :***

Let us consider the case of a body of mass '*m*' at a height '*h*' above the ground. Its kinetic energy is zero, while potential energy is *mgh*.

At position 'A' K.E. = 0, P.E. = *mgh*

The sum of K.E. & P.E. at 'A' is $mgh + 0 = mgh$ . . . . . . (1)

Now, suppose that the body falls at distance x, where the velocity of the body becomes v.

Using the formula $v^2 = u^2 + 2gh$, we have $v = \sqrt{2gx}$ because, $u = 0$ & $h = x$.

In this case, at position 'B' Kinetic Energy of the body is $\frac{1}{2} mv^2 = \frac{1}{2} m.\ 2gx = mgx$

Potential Energy at position B = $mg(h - x)$

The sum of K.E. & P.E. at position 'B' is $mgx + mg(h - x) = mgh$ . . . . (2)

Suppose that body reaches the ground at position 'C' where its potential energy is zero &

K.E. = $\frac{1}{2} mv^2 = \frac{1}{2} m(2gh) = mgh$

Thus the sum of P.E. & K.E. at ground level is $0 + mgh = mgh$ . . . . (3)

From the above equations it is clear that, the sum of P.E. & K.E. at all conditions is constant.

So, the principle of conservation of energy may be stated as ***" the total energy in any system always remain constant ".***

- ***CONSERVATION OF LINEAR MOMENTUM :***

The law of conservation of momentum may be stated as " when no external forces act on bodies then the total momentum before impact is equal to the total momentum after impact ".

m_1, u_1 are the mass & velocity of the first ball before impact,
m_2, u_2 are the mass & velocity of the second ball before impact,
m_1, v_1 are the mass & velocity of the first ball after impact,
m_2, v_2 are the mass & velocity of the second ball after impact.

Momentum before impact = $m_1 u_1 + m_2 u_2$
Momentum after impact = $m_1 v_1 + m_2 v_2$

According to conservation of momentum, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

- ***IMPULSE :***

The impulse of a force acting on a body for any time is the product of the force & the ***s*** time during which the force acts on the body.

Let F = force acting on a body of mass 'm'
t = time for which the force acts on the body

Then impulse of the force on the body is given by the product of force & the time of applied force.

i.e. $I = F t$

*Unit of the impulse are **Newton second (Ns)** or **kg m/s***

- ***IMPULSIVE FORCE :***

An impulsive force is a very great force acting on a body for a very short period of time, so that the change in position of the body during the time the force acts on it, may be neglected.

The whole effect of an impulsive force is measured by the impulse (i.e. the change in momentum produced). Examples – *The force with which the bat strikes the ball is called impulsive force.*
- *The force with which the bullet is drives out of the gun, is impulsive force.*
- *The force with which the nail is stroked by the hammer is impulsive force.*

- ***IMPACT :***

The phenomenon of collision of two moving bodies where we have active & reactive forces of very large magnitude acting during a very short interval of time is called *impact*.

- ***COLLISION OF TWO BODIES & VELOCITY OF COLLISION :***

Condition of collision :

Two bodies will collide when,

1. One of the body (at least) is in motion.
2. Two bodies must touch each other.

- ***PHENOMENON OF COLLISION :***

Whenever two elastic bodies collide with each other, the phenomenon of collision takes place as given below -

1. The bodies, immediately after collision, come momentarily to rest.

2. The two bodies tend to compress each other, so long they are compressed to the maximum value.

3. The two bodies attempt to regain their original shape due to their elasticity. This process of regaining the original shape is called restitution.

The time taken by the two bodies on compression, after the instant of collision, is called the time of compression & time for which restitution takes place is called the time of restitution. The sum of the two times of compression & restitution is called time of collision or period of impact.

$$t_C + t_R = T$$

where, t_C = time of compression,

t_R = time of restitution,

T = period of impact.

- ***COLLISION OF ELASTIC BODIES :***

The property of bodies, by virtue of which, the objects rebound, after striking the floor (or some other body) is called elasticity.

Which rebounds to a greater height is said to be more elastic, than that which rebounds to a lesser height. But, if a body does not rebound at all, after its impact, it is called an inelastic body.

Elastic Impact : A phenomenon of collision, between two elastic bodies which occurs in a very small interval of time & during which the two bodies exert on each other relatively large force is called as elastic impact. The magnitudes of the forces & the duration of impact depend on the shapes of the bodies, their velocities & their elastic properties.

Line of Impact : The common normal to the surface in compact during the impact is called the line of impact.

- ***TYPES OF COLLISIONS :***

When two bodies collide with one another, they are said to have an impact. Following are the two types of impact –

1. Direct impact & **2.** Indirect or Oblique impact.

1. Direct Impact : If the velocities of the particles before & after impact are directed along the line of impact, the impact is said to be a *direct impact*.

Consider two bodies A & B having a direct impact.

Let m_1 = mass of the body A,

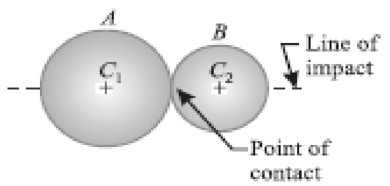
u_1 = initial velocity of the body A before impact,

v_1 = final velocity of the body A after impact.

m_2, u_2, v_2 = corresponding values for the body B.

According to law of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

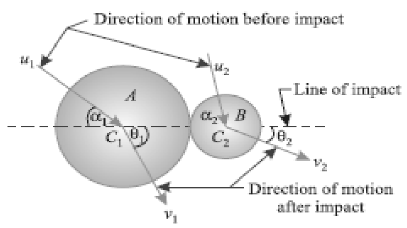


Direct impact

- direction of velocity should always be kept in view while solving examples.
- If velocity of a body is taken as ***+ ve*** in one direction, then the velocity in opposite direction should be taken as ***– ve***.
- If one of the body is initially at rest, then such a collision is also a impact.

2. Indirect or Oblique Impact :

If the two bodies, before impact, are not moving along the line of impact, the collision is called an *indirect (or oblique) impact*. Or If either or both particles move along a line other than the line of impact is said to be an *indirect or oblique impact*.



Indirect impact.

- ***NEWTON'S LAW OF COLLISION OF ELASTIC BODIES :***

It states , " *When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach*."

Mathematically, $(v_2 - v_1) = e\,(u_1 - u_2)$

Where, u_1 = initial velocity of first body,

v_1 = final velocity of first body,

u_2 = initial velocity of second body,

v_2 = final velocity of second body,

e = constant of proportionality = co-efficient of restitution.

$(v_2 - v_1)$ = velocity of separation,

$(u_1 - u_2)$ = velocity of approach.

The co-efficient of restitution (e) = velocity of separation / velocity of approach.

The value of co-efficient of restitution (e) lies between 0 & 1. If e = 0, the two bodies are in elastic. But, if e = 1, the two bodies are perfectly elastic.

- If two bodies are moving in the same direction, before or after impact, then the velocity of approach or separation *is the difference of their velocities*. But if two bodies are moving in the opposite directions, then the velocity of approach or separation *is the algebraic sum of their velocities*.
  - The above formula holds good under the assumed conditions (i.e. $u_1 > u_2$ & $v_2 > v_1$). But if the above formula not holds good, then the formula may be adjusted accordingly, to keep both the sides of the equation as positive.

- ***DIRECT IMPACT OF A BODY WITH A FIXED PLANE :***

Now consider a body having a direct impact on a fixed plane.

Let, u = initial velocity of the body,

v = final velocity of the body, &

e = co-efficient of restitution.

We know that, the fixed plate will not move even after impact. Thus the velocity of approach is equal to (u) & velocity of separation is equal to (v).

According to Newton's law of collision for this type of impact, i.e. $v = e\,u$

- In such cases, we do not apply the principle of momentum (i.e. equating the initial momentum & the final momentum), since the fixed plane has infinite mass.
- If a body is allowed to fall from some height on a floor, then the velocity, with which the body impinges on the floor, should be calculated by the relations of plane as below –

Let, H = height from which the body is allowed to fall.

Therefore, velocity with which the body impinges on the floor, $u = \sqrt{2gH}$

- If a body is first projected upwards from the ground with some initial velocity, it will reach the greatest height & will return to the ground with the same velocity, with which it has projected upwards.

- ***DETERMINATION OF VELOCITY AFTER IMPACT IN ELASTIC COLLISION :***

When the two bodies collide, the total kinetic energy of the bodies after impact may not be equal to that before impact. If the total kinetic energy of two bodies remain the same, both after & before impact the collision is said to be " *perfectly elastic* ". Collision of atomic, nuclear & fundamental particle are examples of elastic collision.

Consider two smooth spheres of masses m_1 & m_2 moving along the line joining their centers with velocities u_1 & u_2 respectively. Let after collision, their velocities become v_1 & v_2 respectively.

As momentum is conserved,

Momentum before collision = momentum after collision.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad . \ . \ . \ . \ . \quad (1)$$

In elastic collision, the total energy remain constant i.e.

Energy before collision = Energy after collision

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad . \ . \ . \ . \ . \quad (2)$$

Dividing equation (2) by equation (1), we have, $u_1 + v_1 = v_2 + u_2$

$$u_1 - u_2 = -(v_1 - v_2) \text{ or } v_1 - v_2 = -(u_1 - u_2) \quad . \ . \ . \ . \ . \ . \ . \quad (3)$$

The velocity v_1 & v_2 may be obtained in the following way from equation (3)

$$v_1 = -u_1 + u_2 + v_2$$

Substituting this value of v_1 in equation (1), we get

$$m_1 (u_1 + u_1 - u_2 - v_2) = m_2 (v_2 - u_2)$$

$$2m_1 u_1 - m_1 u_2 - m_1 v_2 = m_2 v_2 - m_2 u_2$$

$$2m_1 u_1 - m_1 u_2 + m_2 u_2 = m_2 v_2 + m_1 v_2$$

$$2m_1 u_1 + (m_2 - m_1) u_2 = (m_1 + m_2) v_2$$

$$\boldsymbol{v_2 = [2m_1 / (m_1 + m_2)] u_1 + [(m_2 - m_1) / (m_1 + m_2)] u_2} \quad . . . . . \quad (4)$$

Similarly we can obtain the value of v_1,

$v_2 = v_1 + u_1 - u_2$

Substituting this value of v_2 in equation (1), we get

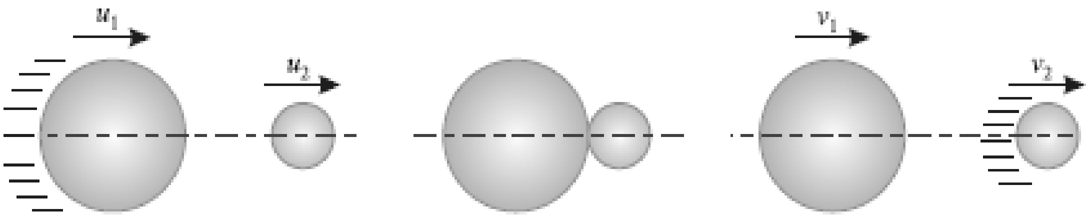
$m_1 (u_1 - v_1) = m_2 (v_1 + u_1 - u_2 - v_1)$

$m_1 u_1 - m_1 v_1 = m_2 u_1 - m_2 u_2$

$m_1 v_1 = m_1 u_1 - m_2 u_1 + m_2 u_2$

$$\boldsymbol{v_1 = [(m_1 - m_2) / m_1] u_1 + (m_2 / m_1) u_2} \quad . . . . . . \quad (5)$$

- **COEFFICIENT OF RESTITUTION :**



(a) Before impact (b) During impact (c) After impact

Consider two bodies A & B having a direct impact as in fig.

Let, u_1 = Initial velocity of the first body,

v_1 = Final velocity of first body, and

u_2, v_2 = Corresponding values for the second body.

A little consideration will show that, the impact will take place if u_1 is greater than u_2. Therefore, the velocity of approach will be equal to $(u_1 - u_2)$. After impact, the separation of the two bodies will take place, only if v_2 is greater than v_1. Therefore the velocity of separation will be equal to $(v_2 - v_1)$. Now as per Newton's Law of Collision of Elastic bodies:

Velocity of separation = *e x* Velocity of approach

$$\boldsymbol{(v_2 - v_1) = e\,(u_1 - u_2)}$$

$$\boldsymbol{e = (v_2 - v_1) / (u_1 - u_2)}$$

Where e is a constant of proportionality, and is called the coefficient of restitution. Its value lies between 0 and 1. It may be noted that if ***e* = 0**, the two bodies are inelastic. But if ***e* = 1**, the two bodies are perfectly elastic.

1. If the two bodies are moving in the same direction, before or after impact, then the velocity of approach or separation is the difference of their velocities. But if the two bodies are moving in the opposite direction, then the velocity of approach or separation is the algebraic sum of their velocities.
2. The above formula holds good under the assumed conditions (i.e. $u_1 > u_2$ & $v_2 > v_1$). But if the above assumptions do not hold good, in an example, then the formula may be adjusted accordingly, to keep both the sides of the equation as positive.

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